Math 302, 3.1 Integer Equivalence Classes and Symmetries, 3.2A The Definition of a Group

Name: _____

Read Section 3.1, Integer Equivalence Classes and Symmetries and the first part of Section 3.2, up to and including Example 3.13.

This reading contains a review of the integers modulo n, a disucssion of symmetries of plane figures, the definition of a group and a few examples.

Reading Questions

1. Make sure you know the definitions of binary operation/law of composition, group, abelian/commutative group, Cayley table.

2. (a) Write out addition and multiplication tables for $\mathbb{Z}_4$.

   (b) Is $\mathbb{Z}_4$ a group with addition as its operation? Is $\mathbb{Z}_4$ a group with multiplication as its operation?

   (c) Write out the multiplication table for $U(4)$, the group of units of $\mathbb{Z}_4$.

3. Reread the paragraph about rigid motions of a rectangle in Section 3.1. If ρ is the rotation by 180°, μ_1 is the reflection across the vertical axis, and μ_2 is the reflection across the horizontal axis, what is $\rho\mu_1$? (Remember that we read the composition of permutations from right to left, so this means: reflect across the vertical axis first, then rotate by 180°.) What is $\rho\mu_2$?

4. Give an example of an abelian group and an example of a nonabelian group.

5. What struck you in this reading? What is still unclear? What remaining questions do you have?