Math 302, 4.1B Subgroups of Cyclic Groups; 4.2 Roots of Unity in $\mathbb{C}$

Name: ______________________________

Read the second part of Section 4.1, starting with the subsection "Subgroups of Cyclic Groups." The proofs of the theorems are worth reading carefully, because they contain some important ideas. If necessary, refresh your memory on the Principal of Well Ordering (Principle 2.6 in Section 2.1) and the Division Algorithm (Section 2.2.)

Skim through Section 4.2, concentrating on "The Circle Group and the Roots of Unity." We spent a lot of time on roots of unity in Abstract Algebra I; the n-th roots of unity provide a nice example of a cyclic group.

Reading Questions

1. $\mathbb{Z}_6$ is a cyclic group of order 6 generated by 1. Use Theorem 4.13 to find the order of each of the elements of $\mathbb{Z}_6$.

   Note: With additive notation, Theorem 4.13 says: "Let $(G, +)$ be a cyclic group of order n, and suppose that $a \in G$ is a generator of the group. If $b = a + \cdots + a$ (k times) $= ka$, then the order of b is n/d where $d = \gcd(n, k)$."

2. $U(7)$ is a cyclic group of order 6 generated by 3. Use Theorem 4.13 to find the order of each of the elements of $U(7)$.

3. Plot the 6th roots of unity in the complex plane. Which of these are primitive?

4. What struck you in this reading? What is still unclear? What remaining questions do you have?