

Name: _____

Read and take notes on Section 1.7, The Affine Cipher. The first main result in this section can be summarized as follows:

Proposition. Let a and b be integers in the range $0, 1, \dots, 25$ with a relatively prime to 26 (guaranteeing the existence of a multiplicative inverse mod 26.) The affine cipher with key (a, b) can be described by the affine encryption function

$$E_{a,b}(x) = (ax + b) \% 26$$

This function has an inverse, the decryption function with decryption key (c, d) , given by the following formula:

$$D_{a,b}(x) = (cx + d) \% 26, \quad \text{where } c = a^{-1} \text{ and } d = -a^{-1}b$$

Reading Questions

1. Look again at the example in the second paragraph, encrypting 'hello how are you.'
 - (a) What is the key for this affine cipher? What is the affine encryption function?
 - (b) Encode first word of the plaintext, 'hello,' using the assignment 'a' is 0, 'b' is 1, etc. This will give you the encoded plaintext for the first word, as a series of five numbers.
 - (c) Encrypt the encoded plaintext using the affine cipher function $E_{3,11}(x)$, i.e. for each number in your code from 1b, multiply by 3, add 11, and reduce modulo 26.

- (d) Decode the ciphertext using the assignment 0 is 'A', 1 is 'B,' etc. You should get the first word of the ciphertext given in the example, 'GXSSB.'
- (e) According to the proposition, the decryption function is $D_{3,11}(x) = (cx + d) \%26$, where c is the inverse of 3 mod 26 and d is $-11c \%26$. Check that $c = 9$ and $d = 5$.
- (f) Decrypt your encoded ciphertext (the list of numbers from 1c) by applying the function $D_{3,11}(x) = (9x + 5) \%26$ to each number. You should recover the encoded plaintext from 1b.
2. (a) Why is a brute-force ciphertext-only attack harder for the affine cipher than for the shift cipher?
- (b) What kind of attack is likely to succeed against an affine cipher?
3. What struck you in this reading? What is still unclear? What remaining questions do you have?