Name: ____

Read and take notes on Section 4.4 Expected Values.

Reading Questions

1. Make sure you understand the meaning of the following terms: random variable, expected value, fair wager, independent random variables and that you are familiar with the properties of expected value described in the propositions.

2. Let X and Y be random variables and c a real constant. Then

   (a) $E(X + Y) \ =$

   (b) $E(cX) \ =$

   (c) For X and Y to be independent means:

   (d) If X and Y are independent, then $E(XY) \ =$

3. Suppose Paul offers you the following wager. You roll a die, and if the result is an even number, he will pay you \$2. If, however, the result is an odd number, you must pay him one dollar for each pip (dot) showing. (So for example, if you roll a one, you pay him \$1.) Is this a fair wager? If not, are the odds in your favor or his?

4. Suppose there are 4 red balls and 6 blue balls in an urn, and we want to know the expected number of red balls to be drawn in 5 trials (replacing whatever ball is drawn in each trial.)

   Let Ω be the set of all possible outcomes of 5 trials. Let X be the random variable given by

   $$X(\omega) \;=\; \text{the number of red balls drawn in event } \omega$$

   We will break X down into the sum of simpler random variables.

   (a) Let X_1 be the random variable given by:

   $$X_1(\omega) \;=\; \begin{cases} 0 & \text{if the first ball drawn in } \omega \text{ is black} \\ 1 & \text{if the first ball drawn in } \omega \text{ is red} \end{cases}$$

   for ω an outcome in Ω. For example, if ω is the event RBRBB, then $X_1(\omega) = 1$. What is $E(X_1)$?

   Similarly define X_2 to tell whether or not the *second* ball drawn is black (0) or red (1), X_3 to tell whether the *third* ball drawn is black or red, etc., up to X_5.

   For example, if ω is the event RBRBB, then

   $$X_1(\omega) = 1, \;\; X_2(\omega) = 0, \;\; X_3(\omega) = 1, \;\; X_4(\omega) = 0, \;\; X_5(\omega) = 0$$

   What is $E(X_i)$, for $1 \leq i \leq 5$?

   (b) Notice that the number of red balls drawn in ω is

   $$X(\omega) \;=\; X_1(\omega) + X_2(\omega) + X_3(\omega) + X_4(\omega) + X_5(\omega)$$

   Use the additive property of expected value to find $E(X)$.

5. What struck you in this reading? What is still unclear? What remaining questions do you have?