Section 1.2

Theorem 1.5 The theorem should say, "Every Pythagorean triple (a, b, c) is similar to a Pythagorean triple of the form $(q^2 - p^2, 2qp, q^2 + p^2)$, where p and q are positive integers with $q > p > (\sqrt{2} - 1)q$."

Exercise 1.19(i) The answer should be $q = 4$, $p = 3$.

Exercise 1.22 Assume also, as part of the set-up, that the point Q is in the first quadrant. At the end of your proof, you may use the following fact: If c^2 is an integer, then either c is an integer or c is irrational.

Exercise 1.31 This is a challenging problem. First show that there are no positive rational numbers x and y so that $x^4 + 1 = y^2$, using Theorem 1.7. To prove that there are no positive rational numbers x and y so that $x^4 - 1 = y^4$, you will need to prove an analogous result to Theorem 1.7, namely that there is no triple (x, y, z) of positive integers with $x^4 - y^4 = z^2$.

Exercise 1.33 Use the fact that 1 and 2 are not congruent numbers.

Theorem 1.9 Near the end of the proof, the sentence beginning with "When we clear denominators ..." should say, "When we clear denominators, we get $a^4 + 2^4c^4 = (ab)^2$, ..."

Theorem 1.11 The phrase "if and only of" should be replaced by "if and only if." Also (as is made clear by the discussion preceding the theorem), the perfect squares in the arithmetic sequence are perfect *rational* squares, namely squares of rational numbers, not necessarily squares of integers.

Section 1.3

How to Think About It, p 34 After the computation, in the second sentence, in which the gcd, 4, is being written as a linear combination of 124 and 1028, the 0 digit is omitted from 1028.

Extra 1.3 Exercise Prove that an integer $m > 1$ is prime if and only if it has no factorization $m = ab$, where $|a| < m$ and $|b| < m$.

Exercise 1.41(i) This is a more general version of the Division Algorithm, which is very useful. Make sure you understand how this version of the Division Algorithm works by trying several examples. For example, try $a = 5$, $b = 23$, then $a = 5$, $b = -23$, and $a = -5$, $b = 23$, and finally $a = -5$, $b = -23$. You can try to prove this general version of the Division Algorithm as a challenge problem.

Exercise 1.46 After proving the "two out of three" rule, deduce the following handy fact: for integers a, b, c, if c is a common divisor of a and b (meaning $c|a$ and $c|b$), then c divides every integer linear combination of a and b, i.e. for any integers s and t, $c|(sa + tb)$.

Exercise 1.47 There are eight parts to this problem; just pick two or three to do. The point of this problem is to help you understand the proof of Theorem 1.19.

Exercise 1.49 Study the proof of Theorem 1.19 and make a similar argument. Define a subset C of I to be the set of positive elements in I, and let d be the smallest element in C. Then prove that all other elements of I are multiples of d, and all multiples of d are elements of I. As in the case of Theorem 1.19, there is also a trivial case, which needs to be treated separately. Note, however, that the set I given in this exercise is not defined *explicitly* but *implicitly.* Instead of being told exactly what is in I (as in the proof of Theorem 1.19), we are given three *properties* of I. We cannot assume anything about what is in I, except what is implied by the three listed properties.

Exercise 1.71 The numbers a, b, and c are real numbers. Modify part (ii) to say, "In proving Corollary 1.35, we need the fact that

$$ab + a\big((-1)c\big) \;=\; ab - ac.$$

Prove this fact."

Section 1.4

Extra 1.4 Problem Prove the uniqueness of multiplicative inverses in $\mathbb{R}$. In other words, prove: Given any real number $a \neq 0$, if there are real numbers b_1 and b_2 with $ab_1 = 1$ and $ab_2 = 1$, then $b_1 = b_2$.

Section 2.1

Proposition 2.7 The proof of (i) is faulty; it shows that $a^{m+n} = a^{m+n}$, which is obviously not what is intended. The first three steps of the proof are fine, but it should finish as follows:
$a^{m-1}a^n a \;=\; a^{m-1}aa^n \;=\; a^m a^n$.

Exercise 2.3 Hint: Use Exercise 1.56 as well as Exercise 1.58.

Exercise 2.4 Modify to say, "If a is positive and $a \neq 1$, give two proofs that

$$1 \,+\, a \,+\, a^2 \,+\, \ldots \,+\, a^n \;=\; \frac{a^{n+1}-1}{a-1}$$

by induction on $n \geq 0$ and by multiplying the left-hand expression by $(a-1)$."

Exercise 2.8 The point of this exercise is to show that the two different ways of defining the factorial of a number are in fact equivalent. In your proof you should use the notation $n!$ to refer to the factorial as defined in the text (page 51), then use induction prove that $n!$ is always equal to $1 \cdot 2 \cdot 3 \cdot \cdots \cdot n$, for $n \geq 1$.

Exercise 2.12(i) See notes on prime factorization, in which the p-adic order $\mathcal{O}_p$ of an integer is defined. Modify the problem to say, "Prove that a positive integer a is a perfect square if and only if for any prime p, $\mathcal{O}_p(a)$ is even, i.e. if $a = p_1^{e_1} \ldots p_n^{e_n}$ with $p_1, \ldots, p_n$ distinct primes and $e_1, \ldots, e_n$ non-negative integers, then all e_i are even."

Exercise 2.15 "When does equality occur?" This means to find a condition on m and n that implies that $\mathcal{O}_p(m+n) = \min\{\mathcal{O}_p(m) + \mathcal{O}_p(n)\}$.

Section 2.2

Lemma 2.23 The formula for $\binom{n}{r}$ should say that $\binom{n}{r} = 1$ if $r = 0$ or $r = n$ (not, as is stated, if $n = 0$ or $n = r$.)

Example 2.27 In the expansion of $(a+b)^4$, the last term should be $+6(ab)^2$, not $-6(ab)^2$. Hence the last term in the expression for $a^4 + b^4$ should be $-6(ab)^2$.

Section 3.1

Exercise 3.2 Modify (i) to say, "Show, for all positive integers n, that the value of i^n is one of $1, i, -1, i$."

Exercise 3.3 Modify (i) to say, "Show, for all positive integers n, that the value of ω^n is one of $1, \omega, \omega^2$."

Section 3.2

Proposition 3.14 At the end of the proof, it is stated that $\sin\theta = \frac{a}{|z|}$, but it should say that $\sin\theta = \frac{b}{|z|}$.

Corollary 3.19 The imaginary unit is missing in the definitions of z and w. The corollary should begin, "If $z = |z|(\cos\alpha + i\sin\alpha)$ and $w = |w|(\cos\beta + i\sin\beta)$, then $z \cdot w = \ldots$"

Exercise 3.21 The point of this exercise is to take advantage of the fact that we have proven that $\mathbb{C}$ satisfies the nine "fundamental properties" of the real numbers discussed in Section 1.4. In particular, the proofs of the relevant results in Section 1.4 can be very easily modified to prove the corresponding results for complex numbers. Use this as an opportunity to write proofs that use only the laws of substitution and the nine fundamental properties, making sure to cite what law or property you use at each step.

Exercise 3.23 The imaginary unit is missing from the formula for $z - \bar{z}$. The exercise should say, "If $z \in \mathbb{C}$ show that $z + \bar{z} = 2(\Re z)$ and $z - \bar{z} = 2(\Im z) \cdot i$."

Exercise 3.26 Nonnegative integer powers of complex numbers are defined in a way analogous to the definition for nonnegative integer powers of real numbers; see page 51. A negative integer power of a nonzero real or complex number is defined as the corresponding positive power of the multiplicative inverse: if $-n$ is a negative integer and $a \neq 0$ is a real or complex number, then $a^{-n} = (a^{-1})^n$.

Exercise 3.39 The sentence should begin "Let $n \geq 0$ be an integer ...".

Exercise 3.41 For (ii), you may use the following fact about polynomials: If $r_1, r_2, \ldots, r_n$ are distinct roots of a degree n polynomial $f(x)$, then $f(x) = C(x - r_1)(x - r_2)\ldots(x - r_n)$ for some constant C. (This follows from induction and Proposition 6.15, stated at the beginning of Section 3.1, on page 82.)

Exercise 3.42 The integer n should be positive, not merely nonnegative. Also, in part (i) of the question, there is unnecessary repitition of the definition of ζ.

Section 3.3

Example 3.31 As stated, the 8^{th} roots of unity are shown in Figure 3.7. Notice that there are eight of them. The four *primitive* 8^{th} roots of unity are listed: $\cos(\frac{2\pi}{8}) + i\sin(\frac{2\pi}{8})$, $\cos(\frac{6\pi}{8}) + i\sin(\frac{6\pi}{8})$, $\cos(\frac{10\pi}{8}) + i\sin(\frac{10\pi}{8})$, and $\cos(\frac{14\pi}{8}) + i\sin(\frac{14\pi}{8})$.

Theorem 3.32(i) The term ζ is missing from the left-hand side of the equation. The equation should be $1 + \zeta + \zeta^2 + \zeta^3 + \cdots + \zeta^{n-1} = 0$. Also, for this to be true, we need $\zeta \neq 1$. The rest of the theorem holds for any nth root of unity ζ, including $\zeta = 1$.

Exercise 3.50 For (i), you may use the following fact: If $r_1, r_2, \ldots, r_n$ are distinct roots of a degree n polynomial $f(x)$, then $f(x) = C(x - r_1)(x - r_2)\ldots(x - r_n)$ for some constant C.

Exercise 3.56 In this exercise you will construct a cubic polynomial with "nice" real coefficients that has three non-obvious real roots. This is similar to Example 3.34, which constructs a quadratic polynomial. Both this exercise and the example use Exercise 3.23 (that the sum of a complex number and its conjugate is real) and Theorem 3.32 (especially that the nth roots of unity sum to zero: make sure you are using the corrected version of this theorem, stated above). Exercise 3.15 will be helpful for the last part of this exercise.