Name: ____

Read Section 4.3, starting with the definition of a commutative ring at the top of page 156 and proceeding up to but not including Units and Fields, page 160.

Reading Questions

1. Write the definition of a commutative ring, word for word, in your notebook. Make sure not to leave out important phrases like, "for all" and "there exists."

2. Following the definition are several examples. Pay particular attention to the examples in Example 4.30(i)-(iii) and (vii)-(vii) and Theorem 4.32. (*Note: We will review the Gaussian integers and Eisenstein integers, for those who have not seen them.*)

3. Reread the first paragraph under Properties of Commutative Rings (page 159), and scan the statements and proofs of the results (namely the propositions, corollaries, and theorem) on the next two pages. Summarize the paragraph in your own words.

4. Carefully reread the two definitions on the top of page 160. In what sense are these notations "hybrid" notations?

5. What struck you in this reading? What is still unclear? What remaining questions do you have?

Name: ____________________

Read the supplemental notes, on $\mathbb{C}$, $\mathbb{Z}[i]$, and $\mathbb{Z}[\omega]$, referencing Chapter 3 as needed, and then read Section 4.3, pages 160-164, Units and Fields.

Reading Questions

1. There are just two definitions in this reading assignment. What are they? Find them and write them in your notebook, word for word.

2. Reread Example 4.40. Are any of these unclear? If so, take the time to verify the examples for yourself in your notebook.

   Pick your favorite prime p, and list the units in $\mathbb{Z}_p$.

   Pick your favorite composite (i.e. not prime) integer m between 10 and 20, and list the units in $\mathbb{Z}_m$.

3. Study the proofs of Proposition 4.39 and Theorem 4.43, making sure to look up the references to previously proven results. Are there any steps that are unclear?

4. Study the proof of Proposition 4.41, making sure to look up references. Are there any steps that are unclear?

   Would a similar proof work for $\mathbb{Z}[\omega]$? (It might help to reread Proposition 3.38, on page 121.)

5. What struck you in this reading? What is still unclear? What remaining questions do you have?

Name: ______________________________

Read Section 4.3, pages 166-167, Subrings and Subfields, along with the supplemental notes on the Subring Criterion. This is a short reading assignment, so take the time to read it carefully.

Reading Questions

1. There are just two definitions in this reading assignment. What are the two terms that are defined? (Write them here.) Write the definitions in your notebook, word for word.

2. Do you ever read the margin notes? There's a good question (a "query") in the margin near the first definition. State and answer the question here.

3. Proposition 4.46 and Proposition 4.48 give necessary and sufficient conditions for a subset of a ring to be a subring and for a subring of a field to be a subfield, respectively. (See the corrected version of Proposition 4.46.) Write these propositions in your notebook, word for word.

   Study the proof of Proposition 4.46 (in the supplemental notes). Do you have any questions about this proof?

4. Summarize the paragraph after the proof of Proposition 4.46 in your own words. Why is the proposition powerful?

5. Example 4.47 is about a Boolean ring. We will not study such rings in detail, but it is good to be aware of them since they are different from many of the other rings we study. The paragraph after the example explains this; summarize that paragraph here in your own words.

6. What struck you in this reading? What is still unclear? What remaining questions do you have?

Name: ____

Read the introduction to Chapter 5 and Section 5.1, pages 191-193, up to but not including the part on fraction fields. Also read the supplemental notes on the structure of the integers.

Reading Questions

1. (a) Write the definition of a domain in your notebook, word for word.

   (b) Sometimes a more explicit version of the definition is useful. Finish the sentence:

   A domain D is a nonzero commutative ring satisfying the following condition:

   if $a, b \in D$ with $a \neq 0$, $b \neq 0$, then ab ____________.

   (c) The contrapositive of the above condition is sometimes more useful in proofs. Finish the sentence:

   A domain D is a nonzero commutative ring satisfying the following condition:

   if $a, b \in D$ with $ab = 0$, then either ________________ or ________________.

   (d) Proposition 5.1 gives another way to characterize a domain. Write the proposition in your notebook, word for word, and read the proof carefully. Do all the steps of the proof make sense?

2. We have now defined three abstract structures: commutative rings, fields, and domains. How are they related? For example, is every commutative ring a domain? (No. Give an example.) Is every field a domain? Is every domain a field? Cite definitions or propositions; give examples.

3. From the supplemental notes on the structure of the integers, what parts are familiar and what parts are less familiar?

4. What struck you in this reading? What is still unclear? What remaining questions do you have?

Name: ____________________________

Finish reading Section 5.1, on fraction fields. This is a short reading; make sure to read it carefully.

Reading Questions

1. The first paragraph under Fraction Fields explains gives an overview and motivation for the discussion that follows. Summarize this paragraph in your own words. (What are we going to do and why?)

2. State the definition of a fraction field. (In order to do this, you need to back track a bit, because there is notation in the definition that is not explained in the definition itself. In particular, what does $[a, b]$ represent? It's an equivalence class, but what's the equivalence relation?)

3. What struck you in this reading? What is still unclear? What remaining questions do you have?

Name: ______________________________

Read Appendix A.1, on functions.

Reading Questions

1. Appendix 1 is a formal treatment of functions. It begins with *motivation*: the author is trying to convince you that a formal notion of function will be useful. Reread the first page of the appendix, and explain, in your own words, why the informal definition given in most calculus books is problematic.

2. The next several pages contain a lot of formal definitions. Reread and pay particular attention to the definitions of: function, equality of functions, well-definedness and single-valuedness, injection, surjection, and bijection. Write these definitions (word for word) in your notebook. Which of these definitions are confusing to you?

3. This appendix also contains examples to illustrate the definitions. Carefully reread the examples that pertain to the definitions that were confusing to you. Take some notes (in your notebook) on the examples that help you to understand the definitions. Give an example or two of your own below.

4. What struck you in this reading? What is still unclear? What remaining questions do you have?

Name: ____________________

In this section, we give a precise and formal definition of a polynomial. It takes quite a bit of work to do this! Read the notes on Section 5.2, referencing the text for the details, as needed, and take notes in your notebook, as appropriate. Then answer the following reading questions.

Reading Questions

1. By plugging in all possible values of x in Z_7, check that the two functions $f(x) = x^7 + 2x - 1$ and $g(x) = 3x + 6$, when considered as functions $\mathbb{Z}_7 \to \mathbb{Z}_7$, are actually the same function.

2. Use the definition of addition of formal power series to compute the following sums.

   (a) $(1, 1, 1, 1, 1, 1, \ldots) \; + \; (2, 0, 2, 0, 2, 0, \ldots)$

   (b) $(2, 0, 0, 0, \ldots) \; + \; (0, 1, 0, 0, \ldots) \; + \; (0, 0, 3, 0, 0, \ldots)$

3. Use the definition of multiplication of formal power series to compute the following products.

   (a) $(0, 0, 0, \ldots) \cdot (1, 2, 3, 4, \ldots)$

   (b) $(1, 0, 0, 0, \ldots) \cdot (1, 2, 3, 4, \ldots)$

(c) $(5,0,0,0,\dots)\ \cdot\ (1,2,3,4,\dots)$

(d) $(0,1,0,0,\dots)\ \cdot\ (0,1,0,0,\dots)$

(e) $(0,1,0,0,\dots)\ \cdot\ (0,0,1,0,\dots)$

4. Check that there are only four functions $\mathbb{Z}_2 \to \mathbb{Z}_2$. Write input-output tables for each one.

5. What struck you in this reading? What is still unclear? What remaining questions do you have?