

Math 1151, Lecture 010, Evaluative Exercise 2

February 18, 2010

Name: _____

Discussion Section: _____

Discussion TA: _____

Seating Section: Left Front Right Front
 Left Back Right Back

You have twenty-five minutes to complete the following five problems, without using your notes, your book, or a calculator.

Sum and Difference Formulas

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Double-angle Formulas

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$\cos(2\theta) = 2 \cos^2 \theta - 1$$

$$\cos(2\theta) = 1 - 2 \sin^2 \theta$$

Half-angle Formulas

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2}$$

$$\cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$$

1. Solve for x :

$$\frac{\pi}{2} = \frac{2}{3} \sin^{-1} x + \frac{\pi}{3}$$

2. Establish the identity:

$$1 - \frac{\sin^2 \theta}{1 - \cos \theta} = -\cos \theta$$

3. If $\cos \alpha = \frac{1}{4}$, $0 < \alpha < \frac{\pi}{2}$ and $\sin \beta = -\frac{3}{5}$, $-\frac{\pi}{2} < \beta < 0$, find the exact value of

(a) $\sin(\alpha + \beta)$

(b) $\cos(2\alpha)$

(c) $\sin\left(\frac{\beta}{2}\right)$

4. Develop a formula for $\cos 3\theta$ as a third degree polynomial in the variable $\cos \theta$. (That means your answer should be in terms of powers of $\cos \theta$, without any $\sin \theta$, $\sin n\theta$, or $\cos n\theta$.)

5. (**Challenge**) Write an equivalent expression for $\cos^4 \theta$ that does not involve any powers of sine or cosine greater than 1.