

**1. Outline of topics**

- Factoring polynomials over the real and complex numbers (4.5, 4.6; see online notes)
- Vectors, projections, dot product (9.4, 9.5)
- Conic sections: parabolas, ellipses, hyperbolas: characteristics, equations, graphs (10.1-10.4)
- Systems of linear equations: 2-var and 3-var systems; types: inconsistent, consistent, independent, dependent; methods: substitution, elimination, matrices (11.1, 11.2)
- Sequences: intro to sequences, arithmetic sequences, geometric sequences (12.1-12.3)

**2. Skills**

You need to be able to:

1. determine the possible rational roots of a polynomial
2. factor a polynomial over the real numbers and over the complex numbers
3. add and subtract vectors, find magnitude, find unit vector
4. compute the projections and dot products of vectors
5. determine characteristics of a conic section from its equation and vice versa
6. graph conic section from its characteristics or its equation
7. solve any 2-var system of linear equations
8. solve 3-var consistent system of independent linear equations
9. solve 3-var inconsistent system of linear equations or 3-var system of dependent equations
10. determine whether a sequence is arithmetic or geometric, find common difference or common ratio
11. find the formula for an arithmetic or geometric sequence
12. compute finite sums of arithmetic and geometric sequences
13. compute infinite sums of convergent geometric sequences

**3. Practice Problems**

- Chapter 4 (p 235): 55, 57, 59, 81, 83, 85
- Chapter 9 (p 617): 59, 61, 63, 69, 85, 87, 99, 100
- Chapter 10 (p 683): 1, 3, 5, 7, 13, 19, 21, 23, 25
- Chapter 11 (p 774): 1, 9, 14, 15, 35, 37, 39, 41
- Chapter 12 (p 820): 13, 17, 21, 25, 27, 29, 31, 33, 37, 39, 41, 43, 45, 47

**4. Basic facts**

The magnitude of a vector  $v = a\hat{i} + b\hat{j}$  is:

$$|v| =$$

The unit vector in the direction of  $v = a\hat{i} + b\hat{j}$  is:

$$\hat{v} =$$

If  $v$  is a vector at the origin, and the angle between  $v$  and the  $x$ -axis is  $\theta$ , then the unit vector in the direction of  $v$  is:

$$\hat{v} =$$

The dot product of two vectors  $v$  and  $w$  with an angle  $\theta$  between them is:

$$v \cdot w =$$

The dot product of  $v = a_1\hat{i} + b_1\hat{j}$  and  $w = a_2\hat{i} + b_2\hat{j}$  is:

$$v \cdot w =$$

Decomposing a vector  $v$  with respect to another vector  $w$ : Write the component vectors  $v_1$  and  $v_2$  of  $v$ , where  $v_1$  is the component of  $v$  that is parallel to  $w$  and  $v_2$  is the component of  $v$  that is orthogonal to  $w$ .

$$v_1 =$$

$$v_2 =$$

A system of linear equations with no solutions is called \_\_\_\_\_.

A system of linear equations with one solution is called \_\_\_\_\_.

A system of linear equations with infinitely many solutions is called \_\_\_\_\_.

**Parabolas:** give the equation and characteristics for the following parabolas:

1. with a vertical directrix, opening right (or left)
  - (a) with vertex at the origin
  - (b) shifted: vertex at  $(h, k)$ .
2. with a horizontal directrix, opening up (or down)
  - (a) with vertex at the origin
  - (b) shifted: vertex at  $(h, k)$ .

	Equation	Vertex	Focus	Directrix
1(a)				
1(b)				
2(a)				
2(b)				

Graph the four parabolas. Label the vertex and the two points on the latus rectum.

**Ellipses:** give the equation and characteristics for the following ellipses:

1. when major axis is horizontal (“wide” ellipse)
  - (a) with center at the origin
  - (b) shifted: center at  $(h, k)$ .
2. when major axis is vertical (“tall” ellipse)
  - (a) with center at the origin
  - (b) shifted: center at  $(h, k)$ .

Note: Make sure to include the equation relating  $a$ ,  $b$ , and  $c$ .

	Equation	Center	Foci	Vertices
1(a)				
1(b)				
2(a)				
2(b)				

Graph the four ellipses. Label the vertices and the “sub-vertices” (the points where the minor axis intersects the ellipse.)

**Hyperbolas:** give the equation and characteristics for the following hyperbolas:

1. when transverse axis is horizontal (hyperbola opens left and right)
  - (a) with center at the origin
  - (b) shifted: center at  $(h, k)$ .
2. when transverse axis is vertical (hyperbola opens up and down)
  - (a) with center at the origin
  - (b) shifted: center at  $(h, k)$ .

Note: Make sure to include the equation relating  $a$ ,  $b$ , and  $c$ .

	Equation	Center	Foci	Vertices	Asymptotes
1(a)					
1(b)					
2(a)					
2(b)					

Graph the four hyperbolas. Label the vertices and the asymptotes.

Gauss' trick: the sum of consecutive integers is:

$$\sum_{k=1}^n k =$$

The recursive formula for an arithmetic sequence with first term  $a_1$  and common difference  $d$  is:

$$a_1 =$$

$$a_n =$$

The general formula for the  $n^{\text{th}}$  term in an arithmetic sequence with first term  $a_1$  and common difference  $d$  is:

$$a_n =$$

Give two formulas for the sum  $S_n$  of the first  $n$  terms of an arithmetic sequence  $\{a_n\}$ :

$$S_n =$$

$$S_n =$$

The recursive formula for a geometric sequence with first term  $a_1$  and common ratio  $r$  is:

$$a_1 =$$

$$a_n =$$

The general formula for the  $n^{\text{th}}$  term in a geometric sequence with first term  $a_1$  and common ratio  $r$  is:

$$a_n =$$

The formula for the sum  $S_n$  of the first  $n$  terms of a geometric sequence  $\{a_n\}$ :

$$S_n =$$

An infinite geometric series converges when \_\_\_\_\_.

The sum of a convergent infinite geometric series is

$$\sum_{k=1}^{\infty} a_1 r^{k-1} =$$