3.1 Derivatives of Polynomials and Exponential Functions

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1. Overview

Outline:

1. The derivative of a constant is zero and the derivative of x is one.
2. Power Rule: $\frac{d}{dx}(x^a) = ax^{a-1}$ for all $a \neq 0$.
3. Constant multiples "slide out": $\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x)$.
4. The derivative of a sum (or difference) is the sum (or difference) of the derivatives.
5. The derivative of e^x is itself.

With these basic facts we can take the derivative of any polynomial function, any exponential function, any root function, and sums and differences of such.

Because taking the derivative of a power of x is easy, it's good to remember how to rewrite fractions and roots in terms of powers:

1. $\frac{1}{x^a} = x^{-a}$
2. $\sqrt[n]{x} = x^{1/n}$

2. Examples

1.) Find the derivative of

$$y = \frac{x^2 - 2\sqrt{x}}{x}$$

We can rewrite y as:

$$y = \frac{x^2}{x} - \frac{2\sqrt{x}}{x} = x - (2x^{1/2})(x^{-1}) = x - 2x^{-1/2}$$

Now we can use the power rule:

$$y' = 1 - 2(-\tfrac{1}{2})(x^{-3/2}) = 1 + x^{-3/2}$$

2.) Find the derivative of

$$y = e^{x+1} + 1$$

We can rewrite y as:

$$y = (e^x)(e^1) + 1 = e(e^x) + 1$$

And e is just a constant number, so it "slides out" when we take the derivative:

$$y' = e(\tfrac{d}{dx}e^x) + 0$$

But the derivative of e^x is itself, so:

$$y' = e(e^x) = e^{x+1}$$

3.) Find all the places where the following curve has a horizontal tangent:

$$y = 2x^3 + 3x^2 - 36x + 17$$

A horizontal line has slope zero, and the slope of the tangent line is the value of the derivative, so y will have a horizontal tangent when $y' = 0$. So we need to take the derivative, set it equal to zero, and solve for x.

Using the power rule, we compute the derivative to be:

$$y' = 6x^2 + 6x - 36 + 0$$

So we want to solve:

$$6x^2 + 6x - 36 = 0$$

We can factor this:

$$6x^2 + 6x - 36 = 6(x^2 + x - 6) = 6(x - 2)(x + 3)$$

So $y' = 0$ when $x = -3$ or $x = 2$. So we can conclude that y has a horizontal tangent when $x = -3$ or $x = 2$.