

4.7 Optimization Problems

Math 1271, TA: Amy DeCelles

1. Overview

Optimization means finding the best possible situation. For example, if you are running a business optimization means minimizing cost or maximizing profit. If you are a physicist, optimization might mean finding the lowest energy state, some kind of equilibrium. These problems become interesting when there is a bit of tension and we have to balance multiple factors. The bit of tension is called the constraint.

To solve an optimization problem follow these steps:

1. What quantity needs to be maximized or minimized? Give that quantity a name.
2. Write that quantity as a function of one variable. Use the constraint to eliminate a variable if necessary.
3. Find the critical numbers.
4. Figure out what interval (domain) makes sense for the function you have. (For example, if your variable is a radius r , it had better not be negative!)
5. Determine the max/min value. If the interval is closed you need to compare the values of the function at critical numbers and endpoints. If the interval is open, you can try the first derivative test.
6. Make sure you answer the question.

2. Examples

- 1.) Find two numbers whose difference is 100 and whose product is a minimum.

What needs to be maximized or minimized. The *product* P needs to be minimized. If the two numbers are x and y , we can say that

$$P = xy$$

We need to eliminate a variable. To do that we use the fact that the difference between the two numbers has to be 100. This is the *constraint*:

$$x - y = 100$$

So we can say $y = x - 100$ and plug that into the equation for P ,

$$P = xy = x(x - 100) = x^2 - 100x$$

Now we find the critical numbers. We need the derivative:

$$P'(x) = 2x - 100$$

The derivative equals zero when $x = 50$ (and there are no places where the derivative does not exist.) Since $x = 50$ is in the domain of $P(x)$ it is a critical number.

Now we have to consider what values of x make sense in the problem. In this problem x could be any real number. So we do not have a closed interval (as in section 4.1). So we use the first derivative test. Notice that when x is less than 50, $P'(x)$ is negative, but when x is greater than 50, $P'(x)$ is positive. That means that there is a local min at $x = 50$. Since $x = 50$ is the *only* critical number, we can conclude that there's actually an absolute min at $x = 50$.

In order to actually answer the question, we need to find y as well.

$$y = x - 100 = 50 - 100 = -50$$

So the two numbers are 50 and -50 .