4.9 Antiderivatives

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1. Overview

Outline

- **Definition** Given a function f, a function F is called an antiderivative of f if $F' = f$.
- **Example** If $f(x) = \cos x + 3x^2$, the function $F(x) = \sin x + x^3 + 17$ is an antiderivative of f.
- **Note** Antiderivatives are not unique. Another antiderivative for $f(x) = \cos x + 3x^2$ would be $F_2(x) = \sin x + x^3 + 21$ and another would be $F_3(x) = \sin x + x^3 + \pi$, ...
- **Plus C** In general, antiderivatieves are determined up to a constant. So if you're asked for "the most general form of the antiderivative" that means you should have a "$+C$" meaning plus any constant number C.
- **Formulas** Know all the antiderivative formulas in the chart on page 341.
- **Graphs** If you are given a graph of $f(x)$, you should be able to sketch a graph of an antiderivative $F(x)$.

Sketching an Antiderivative

1. Key: *y-values* $\longrightarrow$ *slopes*
2. $f(x) = 0 \longrightarrow F(x)$ is flat.
3. $f(x)$ is positive $\longrightarrow F(x)$ is increasing.
4. $f(x)$ is negative $\longrightarrow F(x)$ is decreasing.
5. $f(x) = 5 \longrightarrow F(x)$ has a slope of 5, etc.
6. $f(x)$ increasing $\longrightarrow F(x)$ concave up
7. $f(x)$ decreasing $\longrightarrow F(x)$ concave down

2. Examples

1.) Suppose $f'(x) = 8x^3 + 12x + 3$ and $f(1) = 6$. Find $f(x)$.

We are given the derivative of a function and we need to find the original function. Notice that there will be lots of functions with the given derivative (because you can always add a constant number to the original function and that will not change the derivative.) So we start by finding the general form of the derivative (with a "$+C$" in it) and then use the other piece of information $f(1) = 6$, to find the particular function.

We need to think backwards from the way we've been thinking. In particular, in this example, we need to do the power rule backwards:

$$f(x) = 8(\tfrac{1}{4}x^4) + 12(\tfrac{1}{2}x^2) + 3(x) + C = 2x^4 + 6x^2 + 3x + C$$

Now we also know that $f(1) = 6$. So if we plug that in, we can find C:

$$6 = 2(1)^4 + 6(1)^2 + 3(1) + C = 2 + 6 + 3 + C = 11 + C$$

So we have $6 = 11 + C$, which means that C has to be $C = -5$. So we can conclude:

$$f(x) = 2x^4 + 6x^2 + 3x - 5$$

2.) Find the most general form of the antiderivative of:

$$h(t) = \frac{\sin t}{\cos^2 t}$$

We need to figure out how to rewrite this as something that we recognize as a derivative. We could try:

$$h(t) = \frac{\sin t}{\cos^2 t} = \frac{\sin t}{\cos t} \cdot \frac{1}{\cos t} = \tan t \sec t$$

And this is the derivative of $\sec t$! So we say:

$$H(t) = \sec t + C$$

is the most general form of the antiderivative of $h(t)$.

3.) Find the most general form of the antiderivative of:

$$f(x) = \frac{x^2 + x + 1}{x}$$

Again, we need to figure out how to rewrite this function as something that we recognize as a derivative. We can "undo the common denominator":

$$f(x) = \frac{x^2 + x + 1}{x} = \frac{x^2}{x} + \frac{x}{x} + \frac{1}{x} = x + 1 + \frac{1}{x}$$

Now we can recognize the antiderivative:

$$F(x) = \tfrac{1}{2}x^2 + x + \ln|x| + C$$

(Don't forget the absolute value bars in the antiderivative of $\frac{1}{x}$!)