Some Formulas for Volumes of Revolution Problems

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1. Slices: Disks and Washers

When computing the volume by slices you integrate the crossectional area:

$$\begin{aligned} V &= \int_a^b A(x)\,dx \quad \text{(when axis of rotation is a horizontal line)} \\ V &= \int_a^b A(y)\,dy \quad \text{(when axis of rotation is a vertical line)} \end{aligned}$$

Make sure your limits of integration are x-values when you're integrating with respect to x and y-values when you're integrating with respect to y.

For **disks**, the crossectional area is πr^2, so you have

$$\begin{aligned} V &= \pi \int_a^b [r(x)]^2\,dx \quad \text{(when axis of rotation is a horizontal line)} \\ V &= \pi \int_a^b [r(y)]^2\,dy \quad \text{(when axis of rotation is a vertical line)} \end{aligned}$$

In the special case that (i) the region is the area under a curve $y = f(x)$ from $x = a$ to $x = b$, and (ii) you are rotating about the x-axis:

$$V = \pi \int_a^b [f(x)]^2\,dx$$

For **washers**, the crossectional area is $\pi(R^2 - r^2)$, where R is the outer radius, and r is the inner radius, so you have:

$$\begin{aligned} V &= \pi \int_a^b \left([R(x)]^2 - [r(x)]^2\right) dx \quad \text{(when axis of rotation is a horizontal line)} \\ V &= \pi \int_a^b \left([R(y)]^2 - [r(y)]^2\right) dy \quad \text{(when axis of rotation is a vertical line)} \end{aligned}$$

2. Shells

When computing the volume by cylindrical shells, use the following formulas:

$$\begin{aligned} V &= 2\pi \int_a^b r(x)h(x)\,dx \quad \text{(when axis of rotation is a vertical line)} \\ V &= 2\pi \int_a^b r(y)h(y)\,dy \quad \text{(when axis of rotation is a horizontal line)} \end{aligned}$$

where r is the radius of the cylinder and h is the height. Again, make sure that your limits of integration are x-values when you're integrating with respect to x and y-values when you're integrating with respect to y. Also, make sure you don't double-count the shells.

In the special case that (i) the region is the area under a curve $y = f(x)$ from $x = a$ to $x = b$ and (ii) you are rotating about the y-axis:

$$V = 2\pi \int_a^b x f(x)\, dx$$

3. When to Do What

Sometimes you're given a volume of revolution problem and you're not told which method (i.e. slices or shells) to use. I don't think there's any absolute rule that you can memorize to figure out which one will work better, but (based on general patterns) here's something to try:

Curve is:	Axis of Rotation:	Use:
$y = f(x)$	horizontal	slices
$y = f(x)$	vertical	shells
$x = f(y)$	horizontal	shells
$x = f(y)$	vertical	slices