Math 113, Prep: The Derivative Function (2.8A)

Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Main Points:

1. Definition of derivative function as limit of difference quotients
2. Differentiability
3. Graph of derivative function
4. Higher order derivatives

1. Definition of Derivative Function

Definition: If we have a function $f(x)$ we can define a new function, the *derivative* of f to be:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided this limit exists. We call $f(x)$ an *antiderivative* for $f'(x)$.

Note: This is the same expression we had for $f'(a)$ in the previous section. The only difference is that in the previous section we thought about a and $f'(a)$ as fixed numbers, and now we're thinking of $f'(x)$ as a function depending on x.

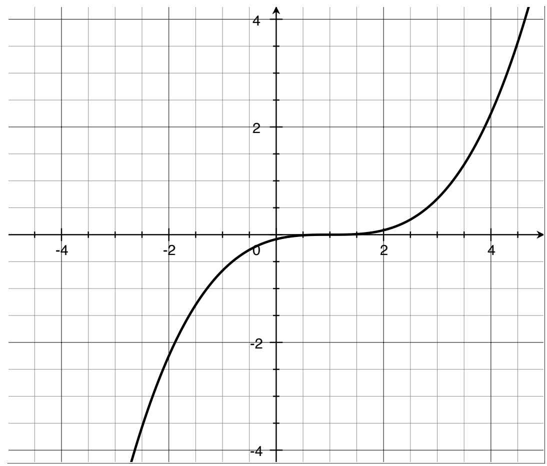
For each x-value in the domain of $f(x)$, the slope of the tangent at that x-value is the value of $f'(x)$. Thus to obtain a graph of $f'(x)$ from $f(x)$ we plot the slopes of $f(x)$. In particular, if the graph of $f(x)$ has a horizontal tangent (slope of zero) at $x = a$, then $f'(a) = 0$, i.e. the graph of $f'(x)$ has an x-intercept at a. If $f(x)$ is increasing at $x = a$, the tangent line has a positive slope, so $f'(a)$ is positive. On the other hand, if $f(x)$ is decreasing at $x = a$, $f'(a)$ is negative. See Example 1, page 155.

Exercises:

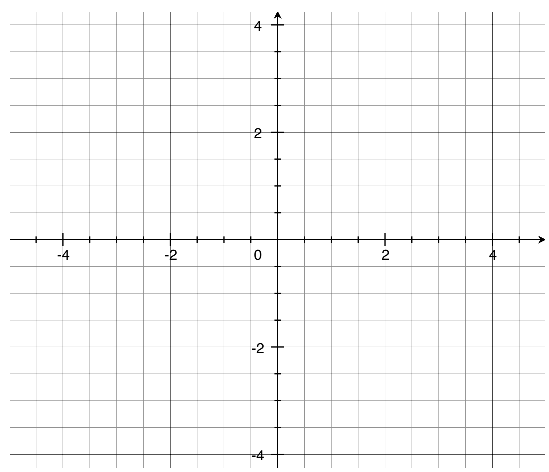
1. Sketch a graph of the function $f(x) = 5$. What is the derivative of this function? In general, what is the derivative of a constant function?

2. Sketch the graph of the function $f(x) = x$. What is the derivative of this function?

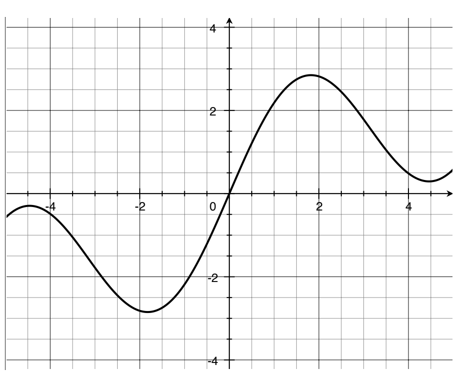
3. The graph of a function $f(x)$ is shown below. Estimate $f'(x)$ for each x-value in the table, by estimating the slope of the tangent line. Plot the points in your table to sketch a graph of the function $f'(x)$.



x	$f'(x)$
-2	
-1	
0	
1	
2	
3	
4	



4. The graph of a function $f(x)$ is below. List the values in ascending order: $f'(-4)$, $f'(-2)$, $f'(0)$, $f'(1)$, $f'(3)$.



2. Differentiability

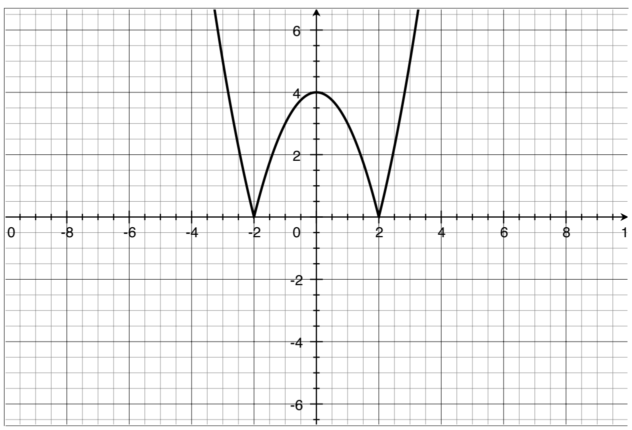
When a function $f(x)$ is not locally linear at a point $x = a$, i.e. when the graph of the function does not look linear even when you zoom in very close, the limit of difference quotients will not exist. (Recall the example of $f(x) = |x^2 - 4|$ and a=2, from the 2.1 and 2.2 prep.) In this case, we say that $f(x)$ is *not differentiable* at $x = a$. This means that the derivative function $f'(x)$ is *undefined* at $x = a$.

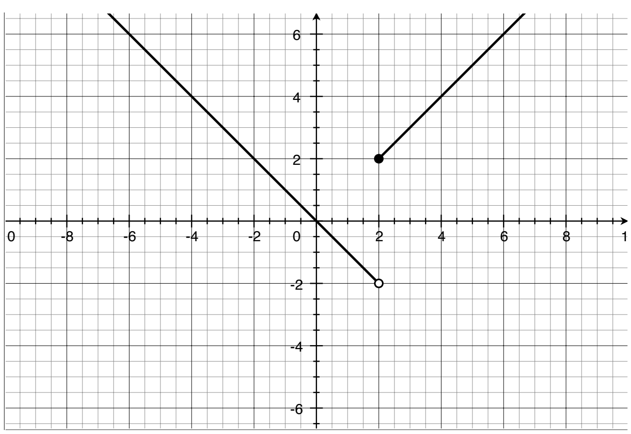
Here are some ways that $f(x)$ could fail to be differentiable (locally linear) at $x = a$: (1) f is discontinuous at a, (2) f has a corner at a, or (3) f has a vertical tangent at a. (See p 159-160.)

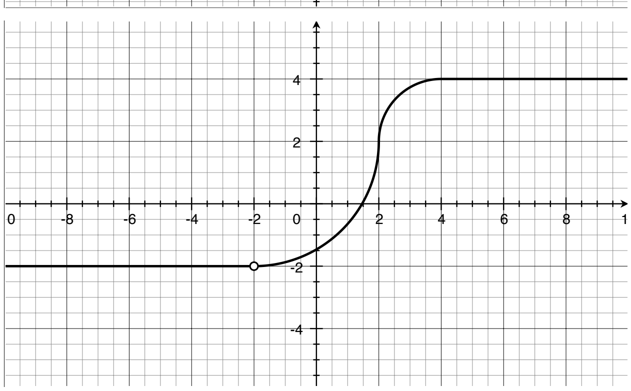
Note: If f is differentiable at a then it is necessarily continuous at a, but not vice versa.

Exercises:

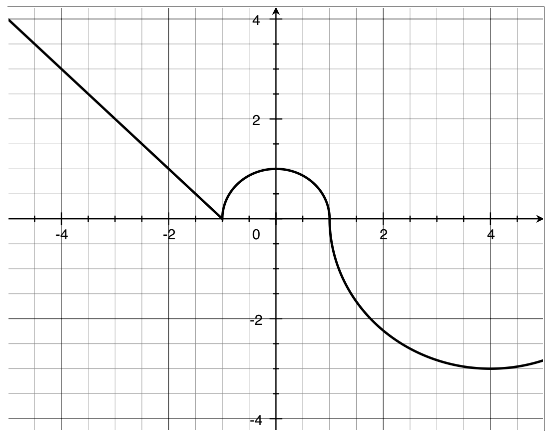
5. For each of the functions below, state, with reasons, the numbers at which the function is not differentiable.



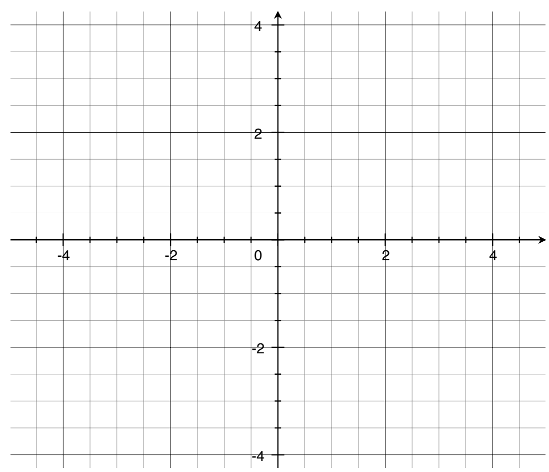




6. The graph of a function $f(x)$ is shown below. Estimate $f'(x)$ for each x-value in the table, and use your table to sketch a graph of the function $f'(x)$.



x	$f'(x)$
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

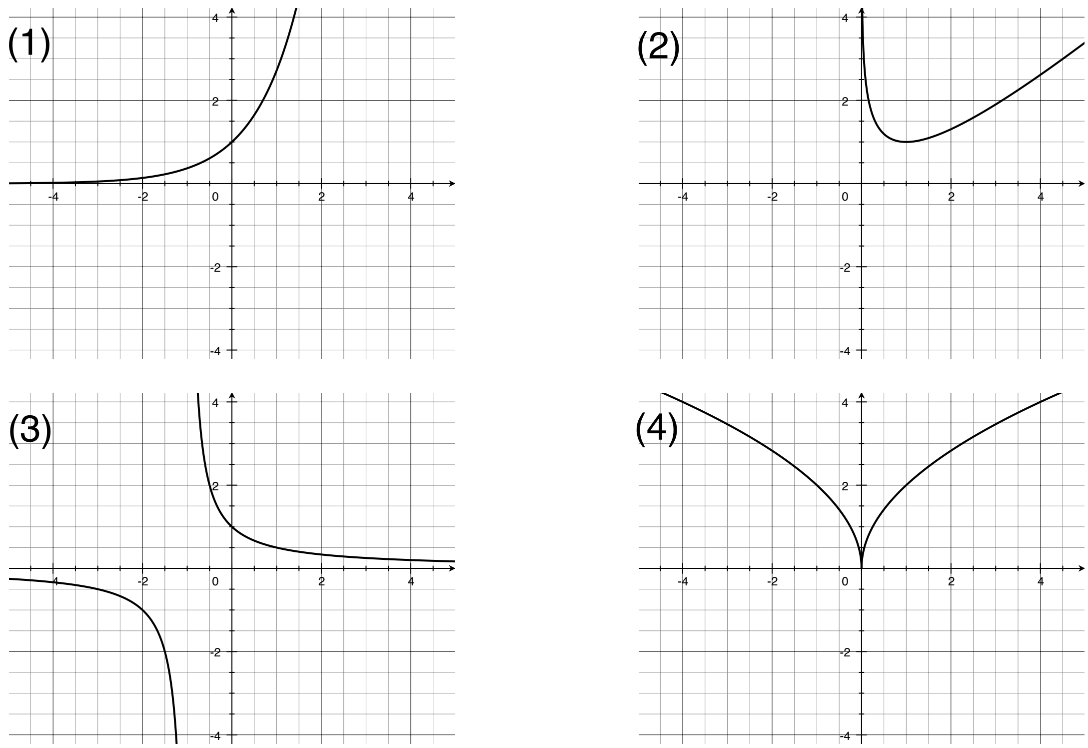


3. Graphs of Derivative Functions: Summary

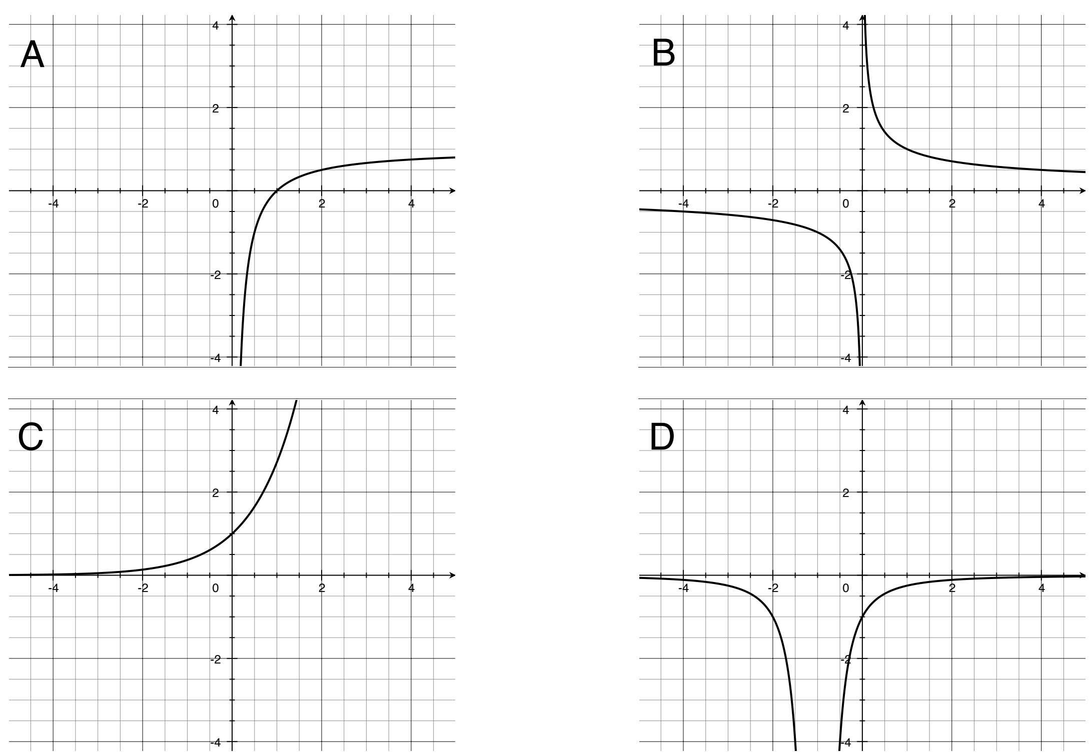
$f(x)$	$f'(x)$
horizontal tangent	x-intercept
discontinuity	undefined
corner	jump discontinuity or vertical asymptote
vertical tangent	vertical asymptote
increasing/decreasing	positive/negative

7. Match the graph of the function with the graph of its derivative.

   Graphs of four functions:



   Graphs of the derivatives of the four functions:



4. Higher Order Derivatives

The derivative of $f'(x)$ is called the second derivative of $f(x)$ and is denoted $f''(x)$. Similarly, the derivative of $f''(x)$ is called the third derivative of $f(x)$ and is denoted $f'''(x)$. Because it is hard to see four or more primes, we do not denote the fourth or higher derivative with primes; instead the fourth derivative is denoted $f^{(4)}(x)$, the fifth derivative denote $f^{(5)}(x)$, etc.

Exercise:

8. The graphs of f, f' and f'' are depicted below. Identify and label each curve, and explain your choices.