Name: ______________________ **Section:** ________

Names of collaborators: ______________________

1. Overview

Increasing and Decreasing: The *first* derivative gives increasing/decreasing information about the original function. (See p 290.)

$f'(x)$ *positive* $\longrightarrow$ slope of the tangent is positive $\longrightarrow$ $f(x)$ is ______________

$f'(x)$ *negative* $\longrightarrow$ slope of the tangent is negative $\longrightarrow$ $f(x)$ is ______________

The only places where f can switch from increasing to decreasing are when $f'(x) = 0$ or $f'(x)$ DNE.

Note: Watch out, these numbers are not necessarily critical numbers, because critical numbers *have to be in the domain of* $f(x)$! For example $f(x) = \frac{1}{x^2}$ switches from increasing to decreasing at $x = 0$, but $x = 0$ is *not* a critical number because it is *not* in the domain of f.

Local Maxima and Minima: Remember from 4.1 that critical numbers are the only possibilities where local max/min may occur. (A local max/min surely must occur at a place in the domain where f switches from increasing to decreasing or decreasing to increasing.) So we find the places where local max/min occur by checking each critical number c. (See p 291.)

$f'(x)$ negative to the left of c, $f'(x)$ positive to the right of c $\longrightarrow$ local _______ at c

$f'(x)$ positive to the left of c, $f'(x)$ negative to the right of c $\longrightarrow$ local _______ at c

If the sign of the derivative is the *same* on both sides of c, then there is neither a local min nor a local max at c. This way of checking the critical numbers is called the *first derivative test*.

Concavity: The *second* derivative gives concavity information about the original function. (See p 293.)

1. $f''(x)$ *positive* $\longrightarrow$ $f(x)$ is concave __________
2. $f''(x)$ *negative* $\longrightarrow$ $f(x)$ is concave __________

The only places where f can switch concavity are when $f''(x) = 0$ or $f''(x)$ DNE.

Inflection Points: A point (x, y) on the graph of $f(x)$ is called an *inflection point* if f is continuous at x and switches concavity at x. (Note that an inflection point is a *point* with an x-value and a y-value.)

Note: Just because a function switches concavity at x, that does not mean it will have an inflection point there. For example, $f(x) = \frac{1}{x}$, $f(x)$ switches concavity at $x = 0$, but $f(x)$ is undefined at $x = 0$, so there is no inflection point there.

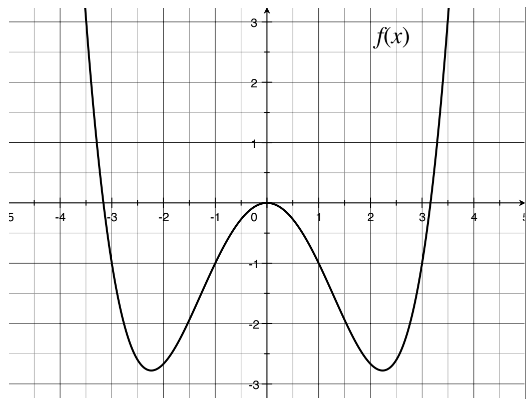
Local Maxima and Minima Revisited: Another way to check a critical number to see if a local max/min occurs there, is by checking concavity instead of increasing/decreasing. This is called the *second derivative test*. You can do this as long as $f''(c)$ exists. (See p 295.)

$f''(c)$ positive $\longrightarrow$ f concave *up* at c $\longrightarrow$ local _______ at c

$f''(c)$ negative $\longrightarrow$ f concave *down* at c $\longrightarrow$ local _______ at c

2. Exercises

1. Consider a function $f(x)$ whose graph is below.



(a) On what intervals is $f(x)$ increasing?

(b) At what values of x does f have a local maximum or minimum?

(c) On what intervals is $f(x)$ concave upward?

(d) What are the x-coordinates of the inflection points of $f(x)$?

2. Consider a function $F(x)$ whose *derivative* is graphed above, in Exercise 1, i.e. $F'(x) = f(x)$.

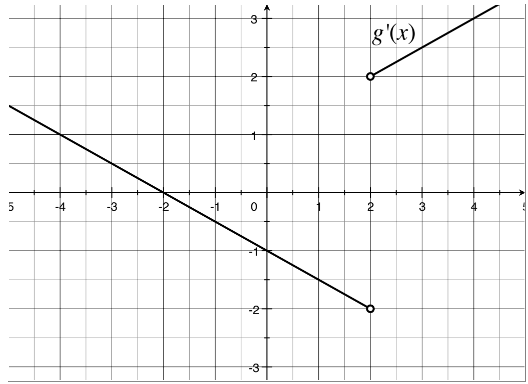
(a) On what intervals is $F(x)$ increasing? Explain.

(b) At what values of x does $F(x)$ have a local maximum or minimum? Explain.

(c) On what intervals is $F(x)$ concave upward? Explain.

(d) What are the x-coordinates of the inflection points of $F(x)$? Why?

3. Suppose $g(x)$ is a continuous function whose derivative is shown below.



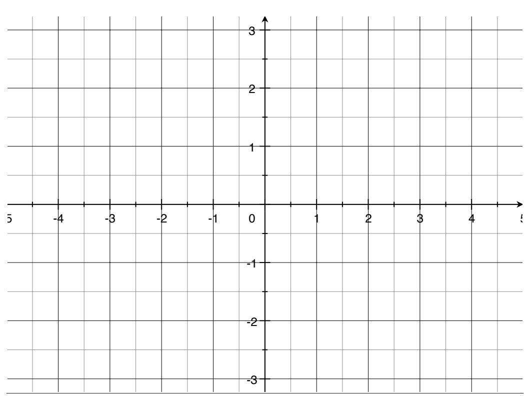
(a) On what intervals is $g(x)$ increasing? Decreasing?

(b) At what values of x does $g(x)$ have a local maximum? Local minimum?

(c) On what intervals is $g(x)$ concave upward? Concave downward?

(d) What are the x-coordinates of the inflection points of $g(x)$?

(e) Assuming that $g(0) = 0$, sketch a graph of $g(x)$ on the axes provided.



4. Consider $f(x) = 5 - 3x^2 + x^3$. Find the intervals of increase and decrease, local maximum and minimum values, the intervals of concavity, and the inflection points. (See Example 6, p 295.)

5. Consider $f(x) = \sin x + \cos x$. Find the intervals of increase and decrease, local maximum and minimum values, the intervals of concavity, and the inflection points.