Math 113, Prep: Derivatives of Inverse Functions and Practicing Differentiation

Name: ______________________ **Section:** ________

Names of collaborators: ______________________

Main Points:

1. Derivatives of logarithmic functions and inverse trig functions
2. Practicing differentiation

1. Derivatives of Inverse Functions

The procedure for finding derivatives of inverse functions will be outlined in the first exercise below.

Exercises

1. In this exercise we will find a formula for the derivative of $f(x) = \ln(x)$. Recall that:

$$f(x) = \ln(x) \quad \Longleftrightarrow \quad e^{f(x)} = x$$

We will differentiate both sides of the second equation: $\frac{d}{dx}\, e^{f(x)} = \frac{d}{dx}\, x$ to get a new equation.

(a) Use the chain rule to differentiate: $\frac{d}{dx}\, e^{f(x)} =$

(b) On the other hand, $\frac{d}{dx}\, x =$

(c) So, if we differentiate both sides of the equation $e^{f(x)} = x$, what new equation do we get?

(d) Now solve for $f'(x)$, to get it on a side by itself.

(e) Now use the fact that $e^{f(x)} = x$ to get a formula for $f'(x)$ in terms of x.

(f) Thus we can conclude: $\frac{d}{dx}\, \ln(x) =$

2. Read page 218, and state the derivative of the logarithm base b:

$\frac{d}{dx}\, \log_b(x) =$

3. Find the derivatives:

   (a) $\frac{d}{dx} \ln(x^2+1) \quad =$

   (b) $\frac{d}{dx} (\log_2(x) - \log_3(x)) \quad =$

   (c) $\frac{d}{dx} x^2 \ln(x) \quad =$

4. Read pages 213-214, and state the derivatives below:

   (a) $\dfrac{d}{dx} \operatorname{arcsin}(x) \quad =$

   (b) $\dfrac{d}{dx} \operatorname{arccos}(x) \quad =$

   (c) $\dfrac{d}{dx} \operatorname{arctan}(x) \quad =$

   (d) $\dfrac{d}{dx} \operatorname{arccot}(x) \quad =$

   (e) $\dfrac{d}{dx} \operatorname{arcsec}(x) \quad =$

   (f) $\dfrac{d}{dx} \operatorname{arccsc}(x) \quad =$

5. Find the derivatives:

   (a) $\frac{d}{dx} \sqrt{\arctan x} \quad =$

   (b) $\frac{d}{dx} x \arcsin x \quad =$

2. Practice with Differentiation

Now we have formulas for all the derivatives of functions in our catalogue of functions, and we have rules for finding derivatives of constant multiples, sums, differences, products, quotients, and compositions of functions. This means we can differentiate extremely complicated functions, as long as we have the patience to do so. For examples of using several differentiation rules on one function see 3.4 Examples 5 & 6 (p 202), and see 3.4 Examples 8 & 9 (p 203) for using the chain rule multiple times on one function.

In particular, when a function is of the form $y = \ln(f(x))$, where $f(x)$ is a complicated function, it is sometimes much easier to use log laws to rewrite y before differentiating. See 3.6 Example 5 (p 219).

Exercises

1. $\frac{d}{dx}\left(\sqrt[3]{x^2} + 11x^5 - x^\pi\right) =$

2. $\frac{d}{dx} \sin^2 5x =$

3. $\frac{d}{dt} e^{-5t}\cos 3t =$

4. $\frac{d}{dx} \ln(x^4 \sin^2 x) =$

5. $\dfrac{d}{dy} \dfrac{(y-1)^4}{(y^2+2y)^5} =$

6. $\dfrac{d}{du}\ \dfrac{u^2 - u}{\sqrt{u}} \quad =$

7. $\frac{d}{dt}\ P(v(t)) \quad =$

8. $\frac{d}{dx}\ f(x)\sec^2(x) \quad =$

9. $\frac{d}{dx}\ (\cot(f(x)) + \sqrt{\pi}) \quad =$

10. $\frac{d}{dw}\ \cos^{-1}(w^2) \quad =$

11. $\frac{d}{dx}\ \ln(\sqrt{x}\cos(x)) \quad =$

12. $\frac{d}{dx}\ 3^{8x-11} \quad =$