Name: ______________________________ **Section:** ________

Names of collaborators: ____________________________________

Main Points:

1. Notions of absolute and local max/min values
2. Definition of critical numbers and procedure for finding absolute max/min values on a closed interval
3. Rolle's Theorem and the Mean Value Theorem

1. Absolute and local max/min values

Read the first half of page 274, up to and including the definition of absolute maximum and minimum values. Note that absolute max/min values refer to the y-values of a function. When we refer to the "location" of a max or min, this is the x-value at which the function has a max/min y-value.

Exercises:

1. (a) Fill in the blanks: According to the text, "[s]ome of the most important applications of differential calculus are ____________________ , in which we are required to find the ____________________ way to do something."

   (b) Give an example (the text lists four) of an application in which finding maximum or minimum values of a function is important.

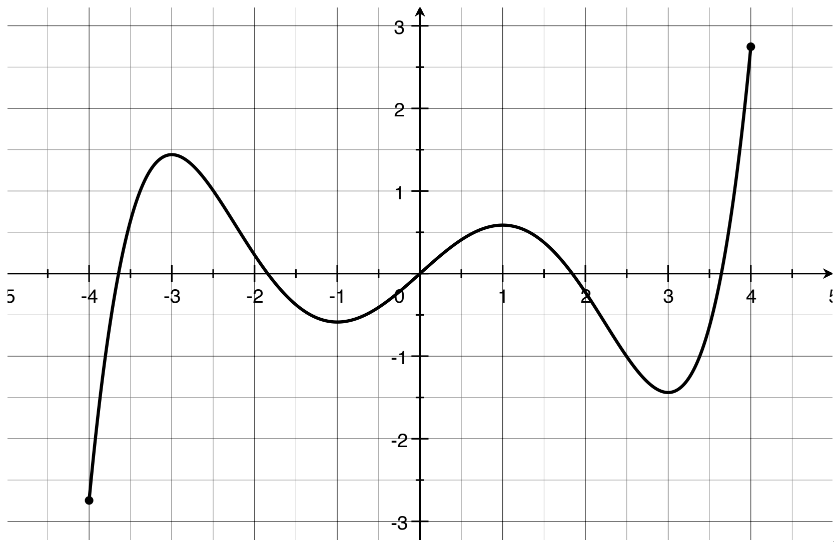
2. What are the absolute maximum value and the absolute minimum value of the function depicted in Figure 1? What are the locations of the absolute maximum and minimum values?

3. What is another name for an absolute maximum? An absolute minimum? What is an extreme value of a function?

Read the second half of page 274, about local maximum and minimum values.

Exercises:

4. Consider the function $f(x)$ whose domain is $D = [-4, 4]$ and whose graph is shown below.

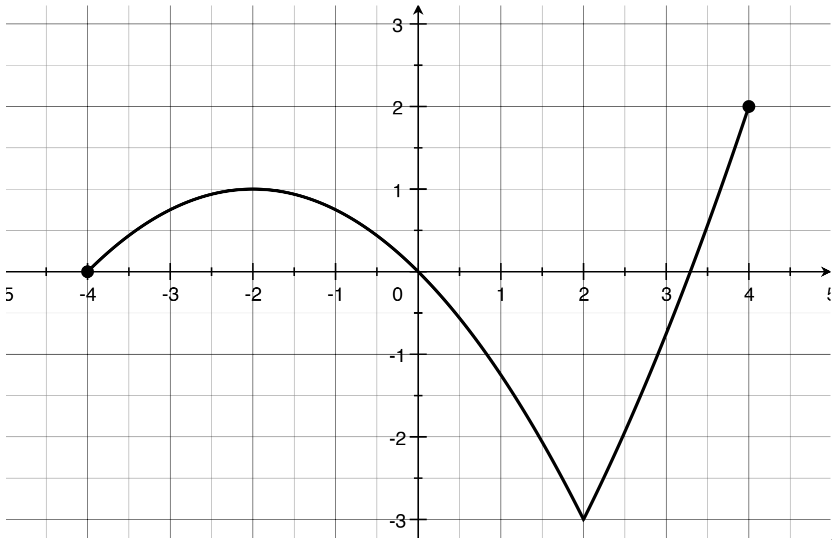


(a) Estimate the absolute maximum value of $f(x)$ and the absolute minimum value of $f(x)$. What are the locations of the absolute max and the absolute min?

(b) Estimate all the local maximum and minimum values of $f(x)$ and give their locations.

(c) For each point on the graph where a local maximum or minimum occurs, describe the derivative of $f(x)$ at that point.

5. Consider the function $g(x)$ whose domain is $D = [-4, 4]$ and whose graph is shown below.



(a) Estimate the absolute maximum value of $g(x)$ and the absolute minimum value of $f(x)$. What are the locations of the absolute max and the absolute min?

(b) Estimate all the local maximum and minimum values of $g(x)$ and give their locations.

(c) For each point on the graph where a local maximum or minimum occurs, describe the derivative of $g(x)$ at that point.

6. Read Examples 1-4 on page 275. Then answer the True/False questions below. Use Examples 1-4 to back up your claims.
   (a) If a function has an absolute maximum value, there can be more than one point on the graph where the maximum value is achieved.

   (b) Every function must have an absolute maximum value and an absolute minimum value.

   (c) Every absolute maximum is also a local maximum.

2. Finding absolute max/min values on a closed interval

The Extreme Value Theorem on page 275 says that every continuous function on a closed interval has an absolute max value and an absolute min value. Further, the absolute max/min values must either occur at a local max/min point or at an endpoint.

Exercise:

7. (a) Look back at the function $f(x)$ in Exercise 4. Does the absolute maximum value occur at a local max/min point or at an endpoint? What about the absolute minimum?

   (b) Look back at the function $g(x)$ in Exercise 5. Does the absolute maximum value occur at a local max/min point or at an endpoint? What about the absolute minimum?

If we want to find the absolute max/min values of a continuous function on a closed interval, we must look at endpoints and local max/min points. But how do we find local max/min points if we do not have a graph of the function? So-called "critical numbers" are important for finding the locations of these local max/min values.

Exercise:

8. (a) State the definition of critical number, which can be found at the bottom of page 277.

   (b) State the critical numbers for the function $f(x)$ in Exercise 4.

   (c) State the critical numbers for the function $g(x)$ in Exercise 5.

   (d) State Theorem 7, on page 278. Check that this is true for the functions $f(x)$ and $g(x)$ in Exercises 4 and 5.

9. The Closed Interval Method (page 278) gives a general procedure for finding max/min values of a continuous function on a closed interval. Check that this method works in the case of the function $g(x)$ in Exercise 5, by going through the three steps of the method and checking that your answer matches your answer from Exercise 5.

3. Rolle's theorem and the MVT

Rolle's Theorem: Suppose f is a function satisfying the following three conditions:

1. f is continuous on a closed interval $[a, b]$
2. f is differentiable on the open interval (a, b)
3. $f(a) = f(b)$

Then we can conclude: There is a number c in (a, b) such that $f'(c) = 0$.

Mean Value Theorem: Suppose f is a function satisfying the following two conditions:

1. f is continous on a closed interval $[a, b]$
2. f is differentiable on the open interval (a, b)

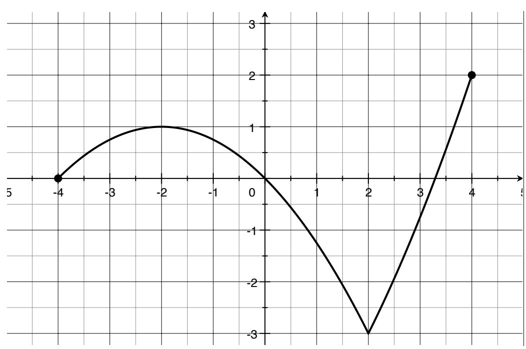
Then we can conclude: There is a number c in (a, b) such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a} \qquad \text{or equivalently,} \qquad f(b) - f(a) = f'(c)(b - a)$$

To put a physical interpretation on this, think of driving a car. I could find my average speed by dividing the distance I traveled by the time it took me. The mean (meaning average!) value theorem says that there was (at least one) moment during my trip when my speed was exactly equal to my average speed.

Exercises:

3. Consider the function $g(x)$ in Exercise 5.



   (a) First we will check to see if $g(x)$ satisfies the hypotheses of Rolle's Theorem on $[-4, 0]$:
      i. Is $g(x)$ continuous on $[-4, 0]$?
      ii. Is $g(x)$ differentiable on $(-4, 0)$?
      iii. Is $g(-4) = g(0)$?

   (b) Now we will check to see if $g(x)$ satisfies the conclusion of Rolle's Theorem on $[-4, 0]$. Is there a number c between -4 and 0 such that when $x = c$, $f'(c) = 0$? (If so, what is it?)

4. Again, consider the function $g(x)$ in Exercise 5.

   (a) First we will check to see if $g(x)$ satisfies the hypotheses of the Mean Value Theorem on $[-4, 2]$:
      i. Is $g(x)$ continuous on $[-4, 2]$?
      ii. Is $g(x)$ differentiable on $(-4, 2)$?

   (b) Now we will check to see if $g(x)$ satisfies the conclusion of the Mean Value Theorem on $[-4, 2]$.
      i. Sketch the secant line through the points $(-4, 0)$ and $(2, -3)$ on the graph above.
      ii. Is there a number c between -4 and 2 such that tangent line at $x = c$ is parallel to the secant line above? (If so, sketch the tangent on the graph above.)