Name: ______________________________ **Section:** ________

Names of collaborators: ______________________________

1. Overview

Optimization means finding the best possible situation. For example, if you are running a business, optimization means minimizing cost or maximizing profit. If you are a physicist, optimization might mean finding the lowest energy state, some kind of equilibrium. These problems become interesting when there is a bit of tension, and we have to balance multiple factors. The bit of tension is called the constraint.

To solve an optimization problem you need to:

1. Determine what quantity needs to be optimized. Give that quantity a name, and write an equation for that quantity in terms of other quantities in the problem. Often it helps to draw a diagram.
2. Write the quantity to be optimized as a function of one variable. Use the constraint to eliminate a variable if necessary.
3. Figure out what interval (domain) makes sense for the function you have. (For example, if your variable is a radius r, it had better not be negative!)
4. Find the critical numbers.
5. Determine the max/min value. If the interval is closed you need to compare the values of the function at critical numbers and endpoints. If the interval is open, you can try the first derivative test.
6. Make sure you answer the question as it was originally posed.

Exercises

1. Look at Example 1 in the textbook and work through the solution yourself by following these steps.
   (a) What quantities are relevant in this problem? Assign variable names to the quantities that vary, and include the units. Which quantity needs to be optimized?

   (b) Draw a diagram illustrating the general situation. Label your diagram with fixed and variable quantities.

(c) Write an equation for the quantity to be optimized in terms of other variables.

(d) Give a sentence explaining why the quantity to be minimized (or maximized) cannot be arbitrarily small (or large, respectively). Write the constraint equation, which relates the variables.

(e) Use the constraint equation to write the quantity to be optimized as a function of one variable.

(f) What is the domain of the function to be optimized?

(g) Find the critical numbers.

(h) Determine the location of the max/min value.

(i) Answer the original question that was posed with a complete sentence. (Make sure to include units.)

2. Find two numbers whose difference is 100 and whose product is a minimum.

   (a) What quantities are relevant in this problem? Assign variable names to the quantities that vary. Which quantity needs to be optimized?

   (b) Write an equation for the quantity to be optimized in terms of other variables.

   (c) Give a sentence explaining why the quantity to be minimized (or maximized) cannot be arbitrarily small (or large, respectively). Write the constraint equation, which relates the variables.

   (d) Use the constraint equation to write the quantity to be optimized as a function of one variable.

   (e) Finish the problem.

3. A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimizes the amount of material used.

   (a) What quantities are relevant in this problem? Assign variable names to the quantities that vary, and include the units. Which quantity needs to be optimized?

   (b) Draw a diagram illustrating the general situation. Label your diagram with fixed and variable quantities.

   (c) Write an equation for the quantity to be optimized in terms of other variables.

   (d) Give a sentence explaining why the quantity to be minimized (or maximized) cannot be arbitrarily small (or large, respectively). Write the constraint equation, which relates the variables.

(e) Use the constraint equation to write the quantity to be optimized as a function of one variable.

(f) Finish the problem.

4. A cone-shaped drinking cup is made from a circular piece of paper of radius 10 cm by cutting out a sector and joining the edges. (See the diagram for Problem 4.7.39.) Cutting out a thin sector results in a wide and shallow cup, whereas cutting out a large sector results in a tall and skinny cup. Find the maximum capacity of a cup made in this way. (Note: the volume of a cone is $V = \frac{\pi}{3}r^2h$, where r is the radius of the top of the cone and h is the height of the cone.)

   (a) What quantities are relevant in this problem? Assign variable names to the quantities that vary, and include units. Which quantity needs to be optimized?

   (b) Draw a diagram illustrating the general situation. Label your diagram with fixed and variable quantities.

   (c) Write an equation for the quantity to be optimized in terms of other variables.

   (d) Give a sentence explaining why the quantity to be minimized (or maximized) cannot be arbitrarily small (or large, respectively). Write the constraint equation, which relates the variables.

(e) Use the constraint equation to write the quantity to be optimized as a function of one variable.

(f) Finish the problem.