Name: _____ **Section:** _____

Names of collaborators: _____

Main Points:

1. The indefinite integral
2. The net change theorem

1. The Indefinite Integral

The indefinite integral is a new notation for an old concept: the family of antiderivatives of a function. For example, the family of antiderivatives of $\cos x$ is $\sin x + C$ where C is any constant. We now notate this in the following way:

$$\int \cos x \, dx \quad = \quad \sin x + C$$

Note: The definite integral is a number; the indefinite integral is a family of functions.

Note: The reasoning behind this notation is that antiderivatives are connected to definite integrals via the fundamental theorem of calculus.

Exercise

1. Evaluate the indefinite integrals:

   (a) $\int 3e^x + 5 \, dx$

   (b) $\int 2x^2 + 5x^{-6/5} \, dx$

   (c) $\int \sec x \tan x - (\sin x)^{-2} \, dx$

(d) $\displaystyle\int \frac{u^2+u+1}{\sqrt{u}}\,du$

(e) $\int w^{-1}(w^3+1)\,dw$

(f) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$

2. The Net Change Theorem

Recall that the accumulated net change in a quatity Q over any time interval $[t_1, t_2]$ can be written as a definite integral of its rate of change $r(t)$:

$$\Delta Q \;=\; \int_{t_1}^{t_2} r(t)\,dt$$

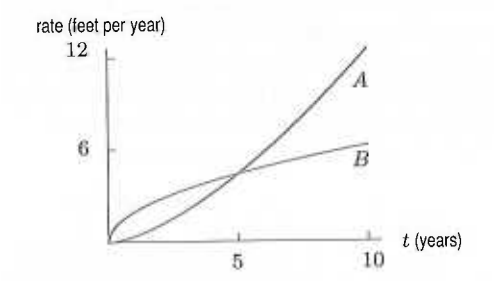
This is the net change theorem.

Exercises

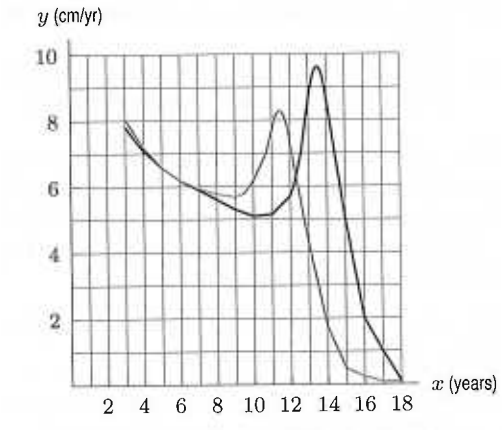
2. (a) If flow rate $r(t)$ has units gallons per hour, and t has units in hours, then what are the units of $\int_0^{100} r(t)\,dt$?

(b) If electricity costs $f(t)$ dollars per day, and t is measured in days, then what does $\int_0^{30} f(t)\,dt$ represent, in terms of electricity?

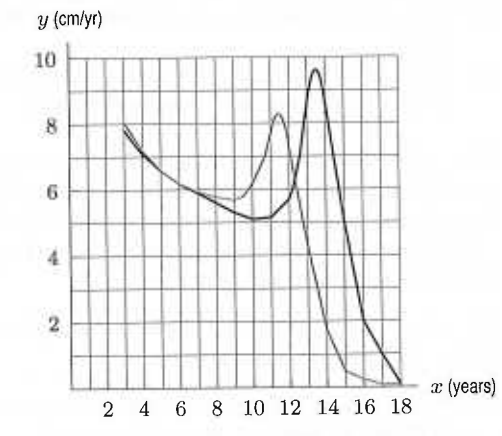
3. The graph below shows the rate of growth of two trees. If the two trees are the same height at time $t = 0$, which tree is taller after 5 years? After 10 years?



4. Height velocity graphs are used by endocrinologists to follow the progress of children with growth deficiencies. The graph below shows the average height velocity curves for boys and for girls between ages 3 and 18. (a) Which curve is for girls and which is for boys? How can you tell?



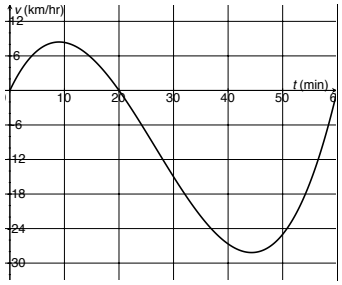
(b) About how much do boys grow between ages 3 & 10?



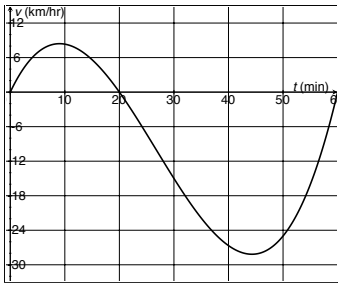
(c) The adolescent growth spurt occurs between ages 12 and 15 for boys and between ages 10 and 12.5 for girls. Estimate the height gained by boys and the height gained by girls during their growth spurt.

(d) When fully grown, about how much taller is the average man than the average woman? (The average boy and the average girl are about the same height at age 3.)

5. A bicyclist is pedaling along a straight road for one hour with a velocity $v(t)$ km/hr shown in the graph below. She starts out five kilometers from the lake, and positive velocities take her toward the lake. (Note: the vertical lines on the graph are at 10 minute (1/6 hour) intervals.) (a) Does the cyclist ever turn around? If so, at what time(s)?



(*Restated*) A bicyclist is pedaling along a straight road for one hour with a velocity $v(t)$ km/hr shown in the graph below. She starts out five kilometers from the lake, and positive velocities take her toward the lake. (Note: the vertical lines on the graph are at 10 minute (1/6 hour) intervals.)



(b) When is she going fastest? How fast is she going then? Toward the lake or away?

(c) Write an integral to represent the net change in the bicyclist's position (i.e. her displacement) during her ride. (Pay attention to units.)

(d) When is she closest to the lake? Write a formula stating exactly how far she is from the lake at that moment, using an integral.

(e) When is she farthest from the lake? Write a formula stating exactly how far she is from the lake at that moment, using an integral.

(f) Does she ever arrive at the lake? How can you be sure?