Name: _____ **Section:** _____

Names of collaborators: _____

Main Points:

1. Differentiating products and quotients of functions
2. The derivative of sine and cosine

1. The Product Rule and the Quotient Rule

Differentiating products and quotients of functions is not as straightforward as differentiating sums and differences of functions. Assuming that f and g are both differentiable functions, the rules below describe how to differentiate the product and quotient of f and g.

Product Rule: The derivative of a product of two functions is: the derivative of the first, times the second, plus the first times the derivative of the second:

$$(fg)' = f'g + fg'$$

Quotient Rule: The derivative of a quotient of two functions is: the derivative of the top, times the bottom, minus the top times the derivative of the bottom, all over the bottom squared:

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Exercises:

1. Suppose $f(x) = x + 1$ and $g(x) = x^2 - 3$. Find the derivative of the product function fg in two ways:

   (a) using the Product Rule,

   (b) rewriting the product as a sum of power functions and then using the Power Rule.

   (c) Compare your two answers to make sure they are the same.

2. Find the derivative of $f(x) = \dfrac{x^4 - 5x^3 + \sqrt{x}}{x^2}$ in two ways:

   (a) using the Quotient Rule,

   (b) rewriting as a sum of power functions and then using the Power Rule.

   (c) Compare your answers to make sure they are the same.

3. Find the derivative of $g(x) = \sqrt{x}\, e^x$.

4. Find the derivative of $y = \dfrac{\sqrt{x}-1}{\sqrt{x}+1}$.

5. Suppose $f(x)$ is a differentiable function. Find the derivatives of the following functions:

   (a) $y = x^2\, f(x)$

   (b) $y = \dfrac{f(x)}{x^2}$

   (c) $y = \dfrac{x^2}{f(x)}$

   (d) $y = \dfrac{1 + x\, f(x)}{\sqrt{x}}$

2. The Derivatives of Sine and Cosine

Skim through pages 191-194 to find the derivatives of sine and cosine and to find the two helpful limits used in the derivations.

Exercises:

6. State the derivatives and limits:

   (a) $\dfrac{d}{dx}\ \sin(x) \ =$

   (b) $\dfrac{d}{dx}\ \cos(x) \ =$

   (c) $\lim\limits_{x\to 0}\ \dfrac{\sin(x)}{x} \ =$

   (d) $\lim\limits_{x\to 0}\ \dfrac{\cos(x)-1}{x} \ =$

7. Differentiate:

   (a) $\dfrac{d}{dx}\ \sin(x)\cos(x) \ =$

   (b) $\dfrac{d}{dx}\ \dfrac{\sin(x)}{\cos(x)} \ =$

   (c) $\dfrac{d}{dx}\ \dfrac{\cos(x)}{\sin(x)} \ =$

(d) $\dfrac{d}{dx}\ \dfrac{1}{\cos(x)}\ =$

(e) $\dfrac{d}{dx}\ \dfrac{1}{\sin(x)}\ =$

(f) $\dfrac{d}{dx}\ x^2\cos(x)\ =$

(g) $\dfrac{d}{dx}\ \dfrac{1+\sin x}{x+\cos x}\ =$

(h) $\dfrac{d}{dx}\ x\sin x\cos x\ =$