Name: ______________________________ **Section:** ________

Names of collaborators: ______________________________

1. Hand-sketching Graphs

In Section 4.5, we put together all that we know about graphs from algebra, precalculus, and calculus to sketch graphs of functions. When graphing a function, ask yourself the following seven questions:

1. **Domain** What is the domain of the function? Are there any numbers for which $f(x)$ is undefined?
2. **Intercepts** What are the x- and y-intercepts?

   To find the x-intercept, set $y = f(x)$ equal to zero and solve for x.

   To find the y-intercept, plug in $x = 0$.
3. **Symmetry** Is the function even? odd? periodic?

   Even: symmetric across the y-axis: $f(-x) = f(x)$ for all x

   Odd: symmetric about the origin: $f(-x) = -f(x)$ for all x

   Periodic: The function repeats (like $\sin x$, $\cos x$, etc.)
4. **Asymptotes** What are the horizontal and vertical asymptotes? (Use limits!) Slant asymptotes?
5. **Intervals of increase and decrease** Where is $f(x)$ increasing/decreasing? (Look at the first derivative!)
6. **Local maxima and minima** Are there any local max/min values? What are their locations?
7. **Concavity and Inflection Points** Where is $f(x)$ concave up/down? Are there any points (x, y) on the graph where f switches concavity? (Look at the second derivative.)

Exercises

1. Consider the function $f(x) = \dfrac{x}{(x-1)^2}$. (See Example 4.5.1.)

   (a) Domain:

   (b) Symmetry:

   (c) Intercepts:

(d) Asymptotes:

(e) Intervals of increase and decrease:

(f) Local max/min values and locations:

(g) Concavity and inflection points:

(h) Sketch the graph:

2. Consider the function $g(x) = \dfrac{x^2+4}{x}$. (See Example 4.5.6.)

(a) Domain:

(b) Intercepts:

(c) Symmetry:

(d) Asymptotes:

(e) Intervals of increase and decrease:

(f) Local max/min values and locations:

(g) Concavity and inflection points:

(h) Sketch the graph:

2. Using *Mathematica* to Refine Graph Sketches

3. Consider the function $f(x) = x^6 - 10x^5 - 400x^4 + 2500x^3$. (See Example 4.6.1.)

   (a) Using *Mathematica*, graph $f(x)$, $f'(x)$, and $f''(x)$, and estimate the roots (x-intercepts) of each of these polynomials. (You will probably need to zoom in and out to find all the roots.)

(b) Using your graph of $f'(x)$, estimate the intervals of increase and decrease of $f(x)$. Estimate the local max/min values and their locations.

(c) Using your graph of $f''(x)$, estimate the intervals of concavity of $f(x)$. Estimate the inflection points.

(d) Sketch graphs of f that reveal all the important aspects of the curve. You will need to sketch more than one graph.

4. Consider the function $g(x) = 6\sin x - x^2$ on the interval $[-5, 3]$. (See Example 4.6.4.)

   (a) Using *Mathematica*, graph $g(x)$, and draw a rough sketch below.

   (b) Using *Mathematica*, graph $g'(x)$, and use this graph to estimate the intervals of increase and decrease of $g(x)$. Estimate the local max/min values and their locations.

   (c) Using *Mathematica*, graph $g''(x)$, and use this graph to estimate the intervals of concavity of $g(x)$. Estimate the inflection points.

   (d) Sketch graphs of g that reveal all the important aspects of the curve. You will need to sketch more than one graph.