Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Main Points:

1. Derivatives of tangent, cotangent, secant, and cosecant
2. Limits involving sine and cosine
3. The chain rule for derivatives of composite functions
4. Derivatives of exponentials

1. Derivatives of More Trig Functions

Formulas for the derivatives of tangent, cotangent, secant, and cosecant can be found using the derivatives of sine and cosine. The formulas can also be found on page 194.

Exercises:

1. State the derivatives:

   (a) $\dfrac{d}{dx}\,\tan(x) \;=$

   (b) $\dfrac{d}{dx}\,\cot(x) \;=$

   (c) $\dfrac{d}{dx}\,\sec(x) \;=$

   (d) $\dfrac{d}{dx}\,\csc(x) \;=$

2. Find the derivative of $g(x) = 4\sec t + \tan t$.

3. Find the derivative of $h(x) = x^{2/3}\csc x$.

2. Limits involving sine and cosine

Recall the two helpful limits used to find the derivatives of sine and cosine:

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1 \qquad\qquad \lim_{x\to 0} \frac{\cos(x)-1}{x} = 0$$

Read Examples 5 and 6, page 196, to see how to use these two limits to evaluate similar limits.

Exercises

4. Evaluate the following limits:

   (a) $\displaystyle\lim_{x\to 0} \frac{\sin 4x}{\sin 6x}$

   (b) $\displaystyle\lim_{x\to 0} \frac{\cos x - 1}{\sin x}$

   (c) $\displaystyle\lim_{t\to 0} \frac{\sin^2 3t}{t^2}$

3. The Chain Rule

To differentiate a composite function like $F(x) = \cos^2(x)$ or $G(x) = \cos(x^2)$, use the chain rule.

Chain Rule: Take the derivative of the outer function, plug in the inner function, and multiply by the derivative of the inner function.

$$(f(g(x))' = f'(g(x)) \cdot g'(x)$$

Another way to write the chain rule is:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

The derivatives of the example functions mentioned above are $F'(x) = (2\cos x) \cdot (\sin x) = 2\cos x \sin x$ and $G'(x) = (\sin(x^2)) \cdot (2x) = 2x \sin(x^2)$.

Exercises

5. Find the derivative of $f(x) = (1 + x^4)^{2/3}$.

6. Find the derivative of $f(x) = \sqrt[3]{1 + \tan x}$.

7. Find the derivative of $y = 5 + \cos^3 x$.

8. Suppose $f(x)$ is a differentiable function. Find the derivatives of the following functions:

   (a) $y = f(x^2)$

   (b) $y = \sqrt{f(x)}$

   (c) $y = \cot(f(x))$

4. Derivatives of Exponential Functions

We know that the derivative of the natural exponential $y = e^x$ is $\frac{dy}{dx} = e^x$. An exponential with any other base will have a slightly different derivative. See page 203 for an explanation of how we can use the chain rule to differentiate $y = a^x$ for an exponential function with any base $a > 0$.

Exercises:

9. State the derivative: $\dfrac{d}{dx}\, a^x \;=$

10. Differentiate the functions:

    (a) $f(x) \;=\; 2^x$

    (b) $g(x) \;=\; 100 \cdot (1/3)^x$

    (c) $h(x) \;=\; (\sqrt{2})^x$

    (d) $r(t) \;=\; t \cdot 4^t$

11. Provide a numerical approximation for the slope of the tangent line to the given curve at $(0, 1)$.

    (a) $y \;=\; 2^x$

    (b) $y = e^x$

    (c) $y = 3^x$