Name: ______________________ **Section:** ________

Names of collaborators: ______________________

Main Points:

1. absolute, conditional convergence
2. ratio test, root test

1. Absolute Convergence

Given a series $\sum a_n$ with positive and negative terms, we may consider the related series $\sum |a_n|$. It is a non-trivial, but true, fact that if this series converges, then the original series must converge. In this case we say that the original series converges *absolutely.* If a series converges, but not absolutely, we say that it converges *conditionally.*

To recap: a series $\sum a_n$ converges *absolutely* if $\sum |a_n|$ converges, but $\sum a_n$ converges only *conditionally* if $\sum a_n$ converges but $\sum |a_n|$ diverges.

An example of an absolutely convergent series is a geometric series with $-1 < r < 0$, like

$$\sum_{n=1}^{\infty} \left(\frac{-2}{3}\right)^{n-1} = 1 - \frac{2}{3} + \frac{4}{9} - \frac{16}{27} + \dots$$

An example of a conditionally convergent series is the alternating harmonic series:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

Exercises.

1. Consider the series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{-3}{5}\right)^{n-1}$.

   (a) Does this series converge? Why or why not?

   (b) Now consider the related series $\sum |a_n|$ (where a_n are the terms of the original series.) Does this series converge? Explain.

   (c) Is the original series absolutely convergent, conditionally convergent, or divergent?

2. Consider the series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{-6}{5}\right)^{n-1}$.

   (a) Does this series converge? Why or why not?

   (b) Now consider the related series $\sum |a_n|$ (where a_n are the terms of the original series.) Does this series converge? Explain.

   (c) Is the original series absolutely convergent, conditionally convergent, or divergent?

3. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

   (a) Does this series converge? (Use the Alternating Series Test.)

   (b) Now consider the related series $\sum |a_n|$ (where a_n are the terms of the original series.) Does this series converge?

   (c) Is the original series absolutely convergent, conditionally convergent, or divergent?

2. Ratio Test and Root Test

We next study two more convergence tests for series with positive terms: the ratio test and the root test. We can use them to test for absolute convergence.

Exercises

4. State the Ratio Test (top of page 734.)

5. Determine whether the following series converge absolutely, converge conditionally, or diverge.

   (a) $\displaystyle\sum_{n=1}^{\infty} e^{-n}\, n!$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\, \frac{2^n}{n^4}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-10)^n}{n!}$

6. State the Root Test (middle of page 736.)

7. Determine whether the following series converge absolutely, converge conditionally, or diverge.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln(n))^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(4n)}{4^n}$