Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Main Points:

1. alternating series test
2. alternating series estimation theorem

1. Alternating Series Test

All of the convergence tests we have learned so far (integral test, comparison test, limit comparison test) have dealt with series with *positive* terms. In the next section we will learn about two more convergence tests for series with positive terms (ratio test, root test). In the meantime we discuss a convergence test for "alternating" series, i.e. series whose terms alternate between positive and negative numbers.

Some of the geometric series we have discussed are alternating. For example,

$$\sum_{n=1}^{\infty} \left(\frac{-2}{3}\right)^{n-1} = 1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \dots$$

is a geometric series with common ratio $r = -2/3$. Since $|r| = 2/3 < 1$, this alternating series converges.

Exercises.

1. (a) Read page 727. State the Alternating Series Test.

   (b) Read Example 1 on page 729. What is the alternating harmonic series? Does it converge or diverge?

   (c) Read Example 2 on page 729. Why can't we use the Alternating Series Test in this example?

2. A series $\sum a_n$ is *conditionally convergent* if $\sum a_n$ converges but $\sum |a_n|$ does not. It is called *absolutely convergent* if $\sum |a_n|$ converges.

   (a) Is the alternating harmonic series conditionally convergent or absolutely convergent?

   (b) Is the series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{-2}{3}\right)^{n-1}$ (from the intro) conditionally convergent or absolutely convergent?

3. In this exercise, we will use the Alternating Series Test to show that the series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{2n+3}$ converges.

   (a) (**Set-up**) Define the series $\sum a_n$ to be the given series and define the sequence $\{b_n\}$ to be the (positive) absolute values of the terms a_n, i.e $b_n = |a_n|$.

   (b) (**Check Hypotheses**) Check to see whether the sequence $\{b_n\}$ satisfies the two hypotheses of the Alternating Series Test, as follows:

      i. Show that $\{b_n\}$ is a decreasing sequence, i.e. $b_{n+1} \geq b_n$ for all n (or at least for all n sufficiently large.) (Hint: Define a function $f(x)$ that "matches" b_n in the sense that $f(n) = b_n$, and take the derivative of $f(x)$. See Example 3, page 729.)

ii. Show that the sequence b_n approaches zero by taking the limit of b_n as $n \to \infty$.

(c) (**Apply Test and Draw Conclusion**) Use the Alternating Series Test to conclude that the series $\sum a_n$ converges, and state your conclusion in a sentence. Make sure to include the phrase "by the Alternating Series Test" somewhere in your sentence.

4. Consider the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n \cos(\pi/n)$.

(a) Use a calculator or *Mathematica* to find decimal approximations for the first five terms of this series. Do you think this series converges or diverges?

(b) Give a careful argument, using the Alternating Series Test or the Test for Divergence, to prove that the series converges or diverges.

2. Alternating Series Estimation Theorem

Recall that we often estimate the sum s of an infinite series using a partial sum s_n for some large number n: $s \approx s_n$. But, of course, an approximation is not very informative unless it is accompanied by an error estimate. This is why we are interested in estimating the remainders of infinite series. In the case of a convergent alternating series, the remainder is quite easy to estimate.

Exercises

5. Read the paragraph on Estimating Sums (page 730), and copy the Alternating Series Estimation Theorem below. See Figure 1 (page 728) for a picture illustrating why the estimate makes sense.

6. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-0.8)^n}{n!}$.

   (a) Use *Mathematica* to graph the first 10 terms and first 10 partial sums on the same screen. (See the *Mathematica* file on Blackboard for help with plotting sequences.) Sketch your results below.

   (b) Use *Mathematica* to graph the first 100 partial sums. Use your graph to make a rough estimate of the sum of the series.

   $s \approx$ ____________________

   (c) Use the Alternating Series Estimation Theorem to estimate the sum correct to four decimal places. (See Example 4.)