Math 114, Practice Modeling with Differential Equations

Name: Amy **Section:** ______

Names of collaborators: ______

Exercises.

1. (a) Money in a bank account earns interest at a continuous annual rate of 5% times the current balance. Write a differential equation for the balance, B, in the bank account as a function of time, t, in years.

$$\frac{dB}{dt} = 0.05B$$

(b) Radioactive substances decay at a rate proportional to the quantity present. Write a differential equation for the quantity, Q, of a radioactive substance present at time t. Is the constant of proportionality positive or negative?

$$\frac{dQ}{dt} = kQ \qquad (k<0 \text{ constant})$$

(c) A pollutant spilled on the ground decays at a rate of 8% a day. In addition, clean-up crews remove the pollutant at a rate of 30 gallons a day. Write a differential equation for the amount of pollutant, P, in gallons, left after t days.

$$\frac{dP}{dt} = -0.08P - 30$$

(d) Toxins in pesticides can get into the food chain and accumulate in the body. A person consumes 10 micrograms a day of a toxin, ingested throughout the day. The toxin leaves the body at a continuous rate of 3% every day. Write a differential equation for the amount of toxin, A in micrograms, in the person's body as a function of the number of days, t.

$$\frac{dA}{dt} = 10 - 0.03A$$

(e) An early model of the growth of the Wikipedia assumed that every day a constant number, B, of articles are added by dedicated wikipedians and that other articles are created by the general public at a rate proportional to the number of articles already there. Express this model as a differential equation for $N(t)$, the total number of Wikipedia articles t days after January 1, 2001.

$$\frac{dN}{dt} = B + kN \qquad (B, k>0 \text{ constants})$$

2. The air in a room with volume 180 m^3 contains 0.15% carbon dioxide initially. Fresher air with only 0.05% carbon dioxide flows into the room at a rate of 2 m^3/min and the mixed air flows out at the same rate.

(a) Let $C(t)$ be the amount of carbon dioxide in the room at after t minutes. What is $C(0)$?

$$C(0) = 180 \times 0.0015 = \frac{180 \times 15}{10{,}000} = \frac{18 \times 15}{1000} = \frac{9 \times 3}{100} = 0.27 \ m^3$$

(b) Write a differential equation for the amount of carbon dioxide, C, in m^3, in the room after t minutes. (Ask yourself how much carbon dioxide is entering the room each minute, and how much is leaving the room each minute.)

entering : $(2m^3)(0.0005) = 0.001 \ m^3$ (each min)

leaving : $(2m^3)(C/180) = C/90 \ m^3$ (each min)

$$\frac{dC}{dt} = 0.001 - C/90 = \frac{1}{90}(0.09 - C)$$

(c) Use separation of variables to obtain a formula for the general solution to this differential equation.

$$\int \frac{dC}{0.09 - C} = \int \frac{1}{90}\, dt$$

$$-\ln|0.09 - C| = \frac{t}{90} + (\text{const.})$$

$$\ln|0.09 - C| = -t/90 + (\text{const.})$$

$$|0.09 - C| = e^{-t/90 + (\text{const.})}$$

$$0.09 - C = \pm e^{-t/90 + (\text{const.})}$$

$$C = 0.09 \mp e^{-t/90 + (\text{const.})}$$

To make this nicer :

$$C(t) = 0.09 \mp (\text{pos const.})\, e^{-t/90} = 0.09 + (\text{nonzero const.})\, e^{-t/90}$$

Would we still have a soln if the const. was zero? i.e. $C(t) = 0.09$ a soln?

- $\frac{dC}{dt} = \frac{d}{dt}(0.09) = 0$
- $\frac{1}{90}(0.09 - C) = \frac{1}{90}(0) = 0$

$\Big\} = \checkmark$ Yes.

$$\boxed{C(t) = 0.09 + Be^{-t/90}}$$ B is any real number

(d) Given your answer in (a), find a formula for the amount of carbon dioxide in the room after t minutes.

$$C(0) = 0.27 = 0.09 + Be^0$$
$$\Rightarrow 0.27 = 0.09 + B$$
$$\Rightarrow 0.18 = B$$

$$\boxed{C(t) = 0.09 + 0.18e^{-t/90}}$$

(e) Find the percentage of carbon dioxide in the room as a function of time. What happens in the long run?

$$P(t) = \left(C(t)/180\right) \times 100 = \frac{9}{180} + \frac{18}{180}e^{-t/90} = \frac{1}{20} + \frac{1}{10}e^{-t/90}$$

As $t \to \infty$ $P(t) \to 1/20$; The percentage of carbon dioxide approaches 0.05%.