Math 114, Prep 11.9A (Power Series as Functions)

Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Main Points:

1. represent a given rational function with a power function
2. using sigma notation, changing indices

Recall that $\sum_{n=1}^{\infty} x^n$ is a function with domain $(-1, 1)$. We can find a rational function that agrees with the power series on its domain by remembering that the sum of a geometric series with initial term a and common ratio r is $a/(1-r)$. Thus

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$$

We can use this fact to find power series representations of some functions that are similar to this one. For example, with $a = 3$ and $r = 4x$,

$$\frac{3}{1-4x} = \sum_{n=0}^{\infty} 3 \cdot (4x)^n = \sum_{n=0}^{\infty} 3 \cdot 4^n x^n = 3 + 12x + 48x^3 + \dots$$

This series will converge when $|r| < 1$, i.e. when $|4x| < 1$, i.e. when $|x| < 1/4$. Thus the interval of convergence of the power series is $(-1/4, 1/4)$.

See Examples 1, 2, and 3 in the textbook for more examples of how to find a power series representation of a function, using the geometric series.

Exercises.

1. Consider the function $f(x) = \dfrac{2}{1-3x}$.

   (a) Find a and r such that $f(x) = a/(1-r)$.

   (b) Write $f(x)$ as a power series using the sigma notation and in expanded form (with ellipsis.)

   (c) Find the radius and interval of convergence of the power series.

2. Find a power series representation and the radius of convergence for each of the following:

   (a) $g(x) = \dfrac{x}{1+x^2}$

   (b) $h(x) = \dfrac{1}{2-x}$

   (c) $F(x) = \dfrac{5}{1-4x^2}$

(d) $G(x) = \dfrac{x^2}{8 - x^3}$

3. Consider the function $f(x) = \dfrac{2}{1 - 3x}$ from the first exercise.

   (a) Write down the first five partial sum functions $s_0(x)$, $s_1(x)$, $\ldots$, $s_4(x)$.

   (b) Use *Mathematica* to plot $f(x)$ and $s_0(x)$ on the same screen. Then do the same for $f(x)$ and each of the other partial sum functions you found in (a). Sketch your results below. Can you tell from your graphs what the radius of convergence is?