Solutions

- 1:30 pm - 3:30 pm Thurs May 22
- Most problems will be similar to problems on previous exams and quizzes. Study those problems and similar problems.
- There will be one fill-in-the-blank question with 20 blanks.
- No calculators, notes, books, cell phones permitted.
- Bring whatever you need to help yourself concentrate for 2 hrs: watch, water bottle, granola bar ...

Basic Facts and Formulas To Know:

- prereqs: basic trig identities, derivatives and antiderivatives of familiar functions
- integration by parts formula
- convergence/divergence of improper integrals involving $\frac{1}{x^p}$
- hypotheses and conclusions of Comparison Theorem (for improper integrals)
- converting between polar and Cartesian coordinates
- slopes and areas with parametric equations and polar coordinates
- convergence/divergence of geometric sequence
- convergence/divergence of geometric series, formula for sum
- convergence/divergence of p-series
- hypotheses and conclusions of convergence/divergence tests
- Taylor series

Integration Techniques, Applications and Interpretation, Improper Integrals (Ch 6-8)

- Integration Techniques: rewrite integrand using algebra, using trig identities, or using a substitution (simple substitution or trig substitution)
- Application and Interpretation: find average value of a function, interpret integral as "continuous sum" or as net change of a rate of change
- Improper Integrals: use limits to describe improper integrals, direct evaluation using antiderivatives, indirect check for convergence/divergence using the Comparison Theorem

Differential Equations, Parametric Equations, Polar Coordinates (Ch 10)

- Differential Equations: qualitative analysis, interpretation (units, graphs), using separation of variables to find formulas for solutions, using initial values to find particular solutions
- Parametric Equations: eliminate parameter, tangent lines, areas
- Polar Equations: tangent lines, areas

Sequences, Series, and Power Series (Ch 11)

- Sequences: finding the limit of a sequence, using LH if necessary
- Series: limit of terms vs limit of partial sums, tests for convergence/divergence
- Power Series: radius and interval of convergence, finding ps representation for functions using geometric series, differentiation, and antidifferentiation or using Taylor series

Practice Problems for Power Series

1. Find the interval of convergence of the series

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{5^n\, n^2}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{4^n\, n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (x-2)^n}{(n+2)!}$

2. Find power series representations for each of the following functions and state the radius of convergence. Write your answer in two ways: (i) with sigma notation: $\displaystyle\sum_{n=0}^{\infty} c_n\, x^n$ and (ii) by writing out the first four terms followed by ellipsis: $c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots$.

   (a) $f(x) = \dfrac{1}{2+x}$

   (b) $g(x) = \dfrac{1}{(2+x)^2}$

   (c) $h(x) = \dfrac{x^2}{(2+x)^3}$

   (d) $r(x) = \ln(2+x)$

3. Find the fourth-degree Taylor polynomial centered at $x = a$ for the function $f(x)$ where

   (a) $f(x) = \sin(x)$, $a = \pi/2$

   (b) $f(x) = \ln(x)$, $a = 1$

   (c) $f(x) = \dfrac{1}{2+x}$, $a = 1$