Math 114, Prep 10.1 (Parametric Curves)

Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Main Points:

1. plotting parametric curves (including initial, final point, direction)
2. eliminating the parameter
3. the cycloid

We have seen that it is possible to describe some curves with equations like $y = f(x)$ or $x = f(y)$. Other curves are better described with a pair of equations, one for the x-coordinate and one for the y-coordinate, where each equation is in terms of a parameter, usually denoted t. These equations are called **parametric equations**, and the curve they trace out in the xy-plane is called a **parametric curve**.

After the parametric curve is plotted in the plane, the role of t is not visible, but we can indicate the role of t using an arrow to indicate direction. Sometimes it is possible to find a Cartesian equation for the curve (one equation involving x and y but not t) by eliminating the parameter t. See Examples 1, 2, and 3.

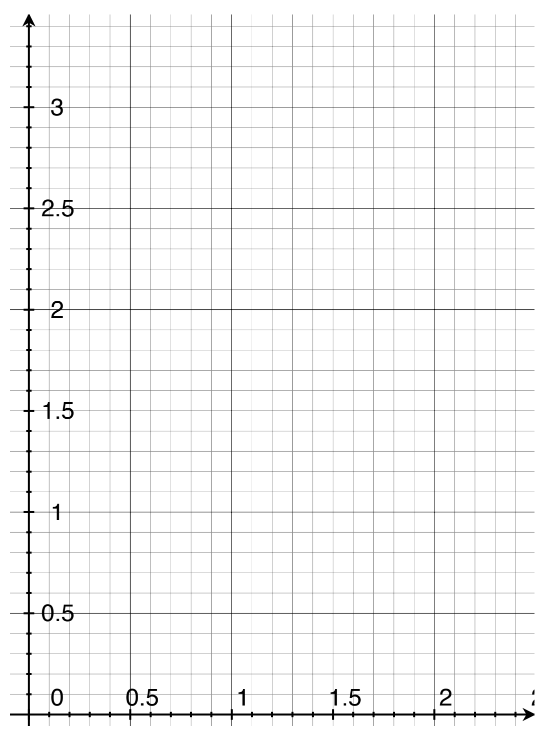
Exercises.

1. Consider the parametric curve given by the following equations:

$$x = \ln(t) \quad y = \sqrt{t} \quad t \geq 1$$

   (a) Make a table of x and y coordinates for the curve. (Round to one decimal place.) Sketch a graph of the curve by plotting the points in your table. Indicate with an arrow the direction in which the curve is traced as t increases.

t	x	y
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		



(b) Eliminate the parameter to find a Cartesian equation of the curve.

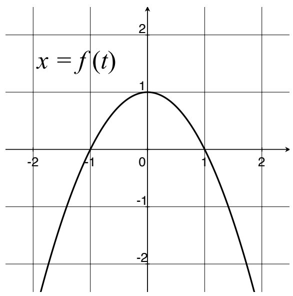
2. Consider the parametric curve given by the following equations:

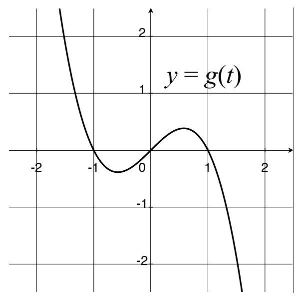
$$x = t - 1 \qquad y = t^3 + 1 \qquad -2 \leq t \leq 2$$

(a) Sketch a graph of the curve by using the parametric equtions to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

(b) Eliminate the parameter to find a Cartesian equation of the curve.

3. Use the graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve $x = f(t), y = g(t)$. Indicate with arrows the direction in which the curve is traces as t increases.





One classical example of a parametric curve is the **cyloid**, which can be described as the path that a reflector on a bicycle wheel traces out as the bike moves along the street. Read Example 7.

4. (a) What are the parametric equations for the cycloid?

   (b) Make a table of x and y values for the cycloid with $\theta = 0, \pi/2, \pi, 3\pi/2, 2\pi$, and use these points to plot the parametric curve. (Your x and y values will be in terms of the radius of the wheel, r.)