Math 114, Prep 10.3 (Polar Coordinates)

Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Main Points:

1. polar coordinates, converting from Cartesian coordinates to polar and vice versa
2. sketching curves by first finding Cartesian equation of curve or by first sketching r as a function of θ
3. slope of tangent

1. Polar coordinates

The xy-coordinates that we are familiar with are called Cartesian coordinates. Some curves are more easily described in terms of distance from the origin and angle with a fixed axis. Read the first few paragraphs of the section as well as Example 1 for a description of polar coordinates.

Examples 2 and 3 show how to convert back and forth from polar to Cartesian coordinates.

Exercises.

1. Plot the point whose polar coordinates are given. Then find two other pairs of polar coordinates of this point, one with $r > 0$ and one with $r < 0$.
   (a) $(2, \pi/3)$

   (b) $(1, -3\pi/4)$

2. What equations do we use to find the Cartesian coordinates of a point whose polar coordinates are known? What equations do we use to find the polar coordinates if the Cartesian coordinates are known? (See the bottom of p 655 and the top of page 656.)

3. Plot the point whose polar coordinates are given. Then find the Cartesian coordinates of the point.

   (a) $(2, -2\pi/3)$

   (b) $(-2, 3\pi/4)$

4. The Cartesian coordinates of a point are $(-1, \sqrt{3})$.

   (a) Find polar coordinates (r, θ) of this point, where $r > 0$ and $0 \leq \theta < 2\pi$.

   (b) Find polar coordinates (r, θ) of this point, where $r < 0$ and $0 \leq \theta < 2\pi$.

5. Sketch the region in the plane consisting of points whose polar coordinates satisfy the given conditions:

   (a) $r \geq 1$

(b) $1 \le r < 2, \quad \pi \le \theta \le 3\pi/2.$

2. Curves in Polar Coordinates

As mentioned above, the equations for certain curves (for example circles!) are much simpler in polar coordinates than in Cartesian coordinates. See Examples 5 and 6. See Examples 7 and 8 for examples of polar curves that are less trivial.

Exercises.

6. Consider the polar curve $r = \sin\theta$.

   (a) Sketch the graph of r as a function of θ in Cartesian coordinates. For what θ-values is r increasing? decreasing? Sketch a rough graph of the polar curve. (See Example 7.)

   (b) Make a table of points on the polar curve (with at least nine θ-values). Plot these points to obtain a more precise graph of the polar curve. (See Example 6.)

   (c) Find a Cartesian equation for the curve. Do you recognize the curve that this equation describes?

3. Tangents to Polar Curves

To find the slope of a tangent line, rewrite the polar equation as a pair of parametric equations, and use the methods of 10.2.

In Example 9, the polar curve is $r = 1 + \sin\theta$. We remember that $x = r\cos\theta$ and $y = r\sin\theta$. Substituting in $r = 1 + \sin\theta$ we obtain:

$$x(\theta) \;=\; (1 + \sin\theta)\cos\theta \qquad y(\theta) \;=\; (1 + \sin\theta)\sin\theta$$

Now we have written the curve in parametric form instead of polar form, and by the methods of 10.2:

$$\frac{dy}{dx} \;=\; \frac{y'(\theta)}{x'(\theta)} \;=\; \frac{\frac{d}{d\theta}(1 + \sin\theta)\cos\theta}{\frac{d}{d\theta}(1 + \sin\theta)\sin\theta} \;=\; \cdots$$

Read Example 9 to see how to finish this problem.

Exercises.

7. Find the slope of the tangent line to the polar curve $r = 2\sin\theta$ at the point specified by $\theta = \pi/6$.