Name: _____ **Section:** _____

Names of collaborators: _____

Main Points:

1. finding formulas for solutions of separable differential equations
2. modeling with differential equations

1. Solving Separable Differential Equations

A separable differential equation is one that can be written in the form: $y' = f(x)g(y)$. For example:

$$y' = xy^3 \qquad y' = x^2\, y^{-2} \qquad y' = 6x^2/(2y + \cos y)$$

are separable differential equations. To solve a separable differential equation, write the derivative in Leibnitz notation (dy/dx instead of y'), write the differential equation in "differential form," and integrate both sides. See Examples 1, 2, and 3.

Exercises.

1. Solve the differential equation $\dfrac{dy}{dx} = \dfrac{e^{2x}}{4y^3}$.

2. Solve the differential equation $y' = xy^2$.

2. Modeling with Differential Equations

Mixing problems and other phenomena can be modeled with differential equations. See Example 6.

Exercises

3. Morphine is administered to a patient intravenously. Let M be the amount of morphine in a patient's bloodstream t hours after the IV is inserted.

   (a) The rate at which the the amount of morphine in the body changes can be represented by the derivative of what quantity, with respect to what quantity? Write the derivative in Leibnitz' notation ("fraction" notation.)

   (b) The rate at which morphine is being administered is 2.5 mg per hour. About 34.7% of the morphine is metabolized and leaves the body each hour. Write a differential equation expressing the rate at which the the amount of morphine in the body changes, as it depends on the amount of morphine currently in the body.

   (c) Use the technique of separation of variables to find formulas for the mathematical solutions of $M(t)$. What happens to M as $t \to \infty$?

Now we will do a *qualitative analysis* of the differential equation. Answer the questions by looking at the differential equation instead of the formulas for solutions. In your answers for (d), (e), and (f), make sure to include all mathematical possibilities, not merely those answers that have a meaningful interpretation in terms of morphine levels.

You might find it helpful to recopy the differential equation here.

(d) What are the constant (equilibrium) solution(s) for this equation? _____

(e) For what values of M is M increasing? _____

(f) For what values of M is M decreasing? _____

(g) For what *meaningful* value(s) of M is M increasing most rapidly? What does this mean in terms of the morphine level in the body?

(h) Sketch four or five mathematical solutions to this differential equation, each with a different initial value. (Make sure to label both axes and provide scale for the M axis.) Which solution curves have a meaningful interpretation in terms of morphine levels?