

Name: _____

Section: _____

Names of collaborators: _____

Main Points:

1. sequences: various notations
2. convergence/divergence of sequences

1. Sequences

A sequence is a list of numbers in a definite order. There are several ways of denoting a sequence, but the simplest involves curly braces and ellipses. Some examples of sequences denoted in this way are:

$$\{1, 2, 3, 4, 5, 6, \dots\} \qquad \{1, -1, 1, -1, 1, -1, \dots\} \qquad \{1, 1/2, 1/4, 1/8, \dots\}$$

Another way of denoting a sequence is by giving a formula for the n^{th} term of the sequence. For example the three sequences above could be represented with the following three formulas:

$$a_n = n, \quad n \geq 1 \qquad b_n = (-1)^n, \quad n \geq 0 \qquad c_n = 1/2^n, \quad n \geq 0$$

See Examples 1 and 2.

Exercises.

1. The formula for the n^{th} term of a sequence is given. Use the formula to find the first five terms of the sequence, and write the sequence in the notation with curly braces and ellipses.

(a) $a_n = \frac{n}{n+1}, \quad n \geq 2$

(b) $b_n = \frac{2n}{n^2+1}, \quad n \geq 0$

(c) $a_n = \frac{(-1)^n n}{n!+1}, \quad n \geq 1$ (Hint: remember that $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$.)

2. Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

(a) $\{\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \dots\}$

(b) $\{4, -7, 10, -13, 16, \dots\}$

(c) $\{6, 9/4, 4/3, 15/16, 18/25, 21/36 \dots\}$

Some sequences are more easily described **recursively**. See Example 3(c) and the beginning of Example 14.

Exercises

3. List the first five terms of the following recursively defined sequences.

(a) $a_1 = 6, \quad a_{n+1} = a_n/n$

(b) $b_0 = 1, \quad b_1 = 2, \quad b_{n+1} = 2b_{n-1} + b_n$

5. Determine whether the following sequences converge or diverge. If convergent, find the limit.

(a) $a_n = \frac{3 + 5n^2}{n + n^2}$ (Hint: divide top and bottom by the highest power of n in the denominator.)

(b) $a_n = e^{2n/(n+2)}$ (Hint: what happens to the “inside function”?)

(c) $a_n = \frac{3^{n+1}}{5^n}$ (Hint: find the common factor.)

(d) $a_n = 2^{-n} \cos(n\pi)$ (Hint: Take the absolute value.)

(e) $a_n = n \sin(1/n)$ (Hint: Find a matching function $f(x)$ and use L'Hospital's rule.)