1 Overview

Main ideas:

1. vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$ (definition as ordered list, geometric description)
2. addition, scalar multiplication (algebraically, geometrically)
3. linear combinations of vectors, the span of a set of vectors
4. vector equations, applications

Examples in text:

1. arithmetic with vectors
2. addition of vectors in $\mathbb{R}^2$, geometrically (parallelogram rule)
3. geometric description of scalar multiples of a vector
4. geometric description of various linear combinations of two vectors
5. determine whether a given vector in $\mathbb{R}^2$ is a linear combination of two specified vectors
6. determine whether a given vector in $\mathbb{R}^3$ is in the span of two specified vectors in $\mathbb{R}^3$
7. using a vector equation in an application (we will omit this for now)

2 Discussion

Goal for 1.3 and 1.4: Develop a matrix-vector viewpoint for linear systems.

Activity: linear combinations worksheet in groups of three

1. compute a linear combination (draw a picture to check!)
2. express a vector as a linear combination of two others (notice: solving a linear system!)
3. impossibility of expressing a vector as a linear combination of two specified vectors (what goes wrong in this example? notion of span)

Main Insight: We can rewrite a linear system as a vector equation, in which the coefficients of the linear system are column vectors and the variables of the linear system are coefficients in the vector equation:

$$\begin{array}{rcrcrcr} 4x & - & 2y & + & z & = & 20 \\ & & 6y & + & 3z & = & 0 \\ -12x & + & 6y & + & 2z & = & -40 \end{array} \qquad x\begin{pmatrix} 4 \\ 0 \\ -12 \end{pmatrix} + y\begin{pmatrix} -2 \\ 6 \\ 6 \end{pmatrix} + z\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 20 \\ 0 \\ -40 \end{pmatrix}$$

Thus, asking whether a given linear system has a solution is equivalent to asking whether a given vector (the rightmost column of the augmented matrix) lies in the span of a specified set of vectors (the other columns of the augmented matrix).