

## Section 1.2

**Theorem 1.5** The theorem should say, “Every Pythagorean triple  $(a, b, c)$  is similar to a Pythagorean triple of the form  $(q^2 - p^2, 2qp, q^2 + p^2)$ , where  $p$  and  $q$  are positive integers with  $q > p > (\sqrt{2} - 1)q$ .”

**Exercise 1.19(i)** The answer should be  $q = 4, p = 3$ .

**Exercise 1.22** Assume also that the point  $Q$  is in the first quadrant.

**Exercise 1.31** This is a challenging problem. First show that there are no positive rational numbers  $x$  and  $y$  so that  $x^4 + 1 = y^2$ , using Theorem 1.7. To prove that there are no positive rational numbers  $x$  and  $y$  so that  $x^4 - 1 = y^4$ , you will need to prove an analogous result to Theorem 1.7, namely that there is no triple  $(x, y, z)$  of positive integers with  $x^4 - y^4 = z^2$ .

**Exercise 1.33** Use the fact that 1 and 2 are not congruent numbers.

**Theorem 1.9** Near the end of the proof, the sentence beginning with “When we clear denominators . . .” should say, “When we clear denominators, we get  $a^4 + 2^4c^4 = (ab)^2, \dots$ ”

**Theorem 1.11** The phrase “if and only of” should be replaced by “if and only if.” Also (as is made clear by the discussion preceding the theorem), the perfect squares in the arithmetic sequence are perfect *rational* squares, namely squares of rational numbers, not necessarily squares of integers.

## Section 1.3

**How to Think About It, p 34** After the computation, in the second sentence, in which the gcd, 4, is being written as a linear combination of 124 and 1028, the 0 digit is omitted from 1028.

**Exercise 1.41(i)** This is a challenging problem. Start by trying several examples. For example, try  $a = 5, b = 23$ , then  $a = 5, b = -23$ , and  $a = -5, b = 23$ , and finally  $a = -5, b = 23$ .

**Exercise 1.49** Study the proof of Theorem 1.19 and make a similar argument. Define a subset  $C$  of  $I$  to be the set of positive elements in  $I$ , and let  $d$  be the smallest element in  $C$ . Then prove that all other elements of  $I$  are multiples of  $d$ . As in the case of Theorem 1.19, there is also a trivial case, which needs to be treated separately.

## Section 2.1

**Proposition 2.7** The proof of (i) is faulty; it shows that  $a^{m+n} = a^{m+n}$ , which is obviously not what is intended. The first three steps of the proof are fine, but it should finish as follows:  
 $a^{m-n}a^n a = a^{m-n}aa^n = a^m a^n$ .

**Exercise 2.4** Modify to say, “If  $a$  is positive and  $a \neq 1$ , give two proofs that

$$1 + a + a^2 + \dots + a^n = \frac{a^{n+1} - 1}{a - 1}$$

by induction on  $n \geq 0$  and by multiplying the left-hand expression by  $(a - 1)$ .”

**Exercise 2.8** The point of this exercise is to show that the two different ways of defining the factorial of a number are in fact equivalent. In your proof you should use the notation  $n!$  to refer to the factorial as defined in the text (page 51), then use induction prove that  $n!$  is always equal to  $1 \cdot 2 \cdot 3 \cdot \dots \cdot n$ .

**Exercise 2.12(i)** Modify the problem to say, “Prove that an integer  $a \geq 2$  is a perfect square if and only if whenever  $p$  is a prime and  $p|a$ , the highest power of  $p$  that divides  $a$  is an even power of  $p$ .”

## Section 2.2

**Lemma 2.23** The formula for  $\binom{n}{r}$  should say that  $\binom{n}{r} = 1$  if  $r = 0$  or  $r = n$  (not, as is stated, if  $n = 0$  or  $n = r$ .)

**Example 2.27** In the expansion of  $(a + b)^4$ , the last term should be  $+6(ab)^2$ , not  $-6(ab)^2$ . Hence the last term in the expression for  $a^4 + b^4$  should be  $-6(ab)^2$ .

## Section 3.2

**Proposition 3.14** At the end of the proof, it is stated that  $\sin \theta = \frac{a}{|z|}$ , but it should say that  $\sin \theta = \frac{b}{|z|}$ .

**Corollary 3.19** The imaginary unit is missing in the definitions of  $z$  and  $w$ . The corollary should begin, “If  $z = |z|(\cos \alpha + i \sin \alpha)$  and  $w = |w|(\cos \beta + i \sin \beta)$ , then  $z \cdot w = \dots$ .”

**Exercise 3.23** The imaginary unit is missing from the formula for  $z - \bar{z}$ . The exercise should say, “If  $z \in \mathbb{C}$  show that  $z + \bar{z} = 2(\Re z)$  and  $z - \bar{z} = 2(\Im z) \cdot i$ .”

**Exercise 3.39** The sentence should begin “Let  $n \geq 0$  be an integer  $\dots$ ”.

**Exercise 3.42** The integer  $n$  should be positive, not merely nonnegative. Also, in part (i) of the question, there is unnecessary repetition of the definition of  $\zeta$ .

## Section 3.3

**Example 3.31** As stated, the 8<sup>th</sup> roots of unity are shown in Figure 3.7. Notice that there are eight of them. The four *primitive* 8<sup>th</sup> roots of unity are listed:  $\cos(\frac{2\pi}{8}) + i \sin(\frac{2\pi}{8})$ ,  $\cos(\frac{6\pi}{8}) + i \sin(\frac{6\pi}{8})$ ,  $\cos(\frac{10\pi}{8}) + i \sin(\frac{10\pi}{8})$ , and  $\cos(\frac{14\pi}{8}) + i \sin(\frac{14\pi}{8})$ .

**Theorem 3.32(i)** The term  $\zeta$  is missing from the left-hand side of the equation. The equation should be  $1 + \zeta + \zeta^2 + \zeta^3 + \dots + \zeta^{n-1} = 0$ . Also, for this to be true, we need  $\zeta \neq 1$ . The rest of the theorem holds for any  $n$ th root of unity  $\zeta$ , including  $\zeta = 1$ .

**Exercise 3.56** In this exercise you will construct a cubic polynomial with “nice” real coefficients that has three non-obvious real roots. This is similar to Example 3.34, which constructs a quadratic polynomial. Both this exercise and the example use Exercise 3.23 (that the sum of a complex number and its conjugate is real) and Theorem 3.32 (especially that the  $n$ th roots of unity sum to zero: make sure you are using the corrected version of this theorem, stated above). Exercise 3.15 will be helpful for the last part of this exercise.