Cultivating an Investigative Mindset in Mathematics for Liberal Arts

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Introduction

MATH 109: Excursions in Mathematics

- Fulfills general education requirement for math.
- Majors in Communications, ASL, Music, Sports Studies ...
- "Mathematical Autobiography" assignment.
  - "bad at math," "not a math person"

How to help them ...

- Experience joy of discovery and understanding?
- Develop confidence to problem-solve and learn?

Hands-on collaborative mathematical investigation!

Teaching about Growth Mindset

- Day 1 Videos: Francis Su (MFHF), Jo Boaler, "Mindset and Practice"
- Article: Carol Dweck, "The Secret to Raising Smart Kids"
- Video: Jo Boaler, "Making Mistakes"
- Video: Jo Boaler, "Thinking Slowly"
- Article: Anne-Laure LeCunff, "From Fixed Mindset to Growth Mindset: The Complete Guide"

Productive Collaboration

Classroom discussion about learning environment:

- What makes a good learning environment?
- What specific things can we do that will help make our classroom a good learning environment for everyone?
- What specific things can "sabotage" the learning environment, making it hard for people to learn in our classroom?

And specifically about productive collaboration:

- What specific words or actions help people work together well in a group?
- What is a helpful thing to say when you disagree? When you think someone has made a mistake? When someone appears frustrated?

Designing In-Class Activities

Goal: collaborative mathematical investigation.

- "Low threshhold, high ceiling."

Practicalities:

- Short introduction.
- Hand-out and supplies (blocks, colored pencils, ...).
- Work in groups of 3-4.

Background for In-Class Activity 1

Strogatz, "The Joy of x"

- Creative aspect of math, within constraints
- Math grows, invention, discovery, freedom
- Playful side of math
- "Rock groups" (Lockhart's Lament)

The Investigative Process:

- Examples
- Conjecture
- Explanation/reasons
- Conclusion

In-Class Activity 1: Rock Groups

"[N]umbers have quirks of structure that endow them with personalities" (Strogatz, 9).

Objectives:

1. Discover "quirks" of numbers by working with them concretely, as "rock rectangles."
2. Practice the process of mathematical investigation.
3. Work collaboratively to investigate a mathematical question.

Questions:

- For a given number of rocks, how many ways are there to arrange the rocks in a rectangular pattern? (Look at the numbers 1-12, at least.)
- What does the number of rectangles tell us about the number of rocks?

Lessons Learned and a Revision

Spring 2023: Chaos

- All the groups had questions, all at the same time.
- Students wanted to know the "right" way to count rectangles.
- Settled for shallow observations.

Spring 2024:

- Discuss "rock group" concepts and model the investigative process the class before the activity.
- Discuss various ways of counting rock rectangles and instruct each group to come to an agreement about how they would count.
- Take time during the class after the activity to debrief and discuss expectations for the report.

Other Investigative Activities

Unbalanced Triangles

The Pythagorean Theorem and Squaring a Sum

Conditional Probabilities

Completing the Square

Estimating Pi and the Area of a Circle

Symmetries of an Equilateral Triangle

Weekly Reflection Questions

Ten prompts, including three directly related to growth mindset:

- Productive Mistake: Describe a moment when you made a mistake and learned from it this week ...
- Again and Again: Describe a situation from class this week in which you persisted in the face of difficulties ...
- Challenge: Describe a situation in class this week where you encountered a challenge and worked through it ...

and one related to collaboration:

- Collaboration: ... What specifically did you do, and what did others do, that helped you work together well.

For more information . . .

My website: `https://www.amydecellesmath.org`

- Archive of Past Teaching
- Excursions in Math, Spring 2024

Thank you for your attention!