

Automorphic Spectral Theory and Number Theoretic Applications

Reed colloquium talk 10/28/2010

A. DeCelles

Innocent-sounding questions about numbers and geometry sometimes require serious mathematics to answer. For example, though the statement of “Fermat’s Last Theorem” is simple enough to discuss with algebra students, the proof required the development of hundreds of years of mathematics, culminating in the proof of Wiles and Taylor in 1995. Spectral theory, breaking down complex functions into fundamental “waves,” in the same way that a prism breaks down white light into its constituent red, orange, yellow, green, blue, and violet light waves, is one modern method for dealing with fundamental questions in number and geometry. As a simple example, Fourier series of Bernoulli polynomials compute special values of Riemann’s zeta function, a function related to the distribution of prime numbers. More serious spectral theory, along with standard methods from complex analysis, gives a relationship between the number of lattice points in an expanding region in hyperbolic space and the automorphic spectrum. The recent work of Diaconu, Garrett, and Goldfeld uses spectral theory to produce an identity involving sums of moments of L-functions, from which they extracted subconvex bounds.